kd education academy (9582701166)

Time : 6 Hour | **STD 11 Science Maths** | **Total Marks : 354**

kd 90+ ch-9 straight line basic to jee

*** Match the following.** **[10]**

1.

Part (A)	Part (B)
1. The angle between the lines $2x - y + 3 = 0$ and $x + 2y + 3 = 0$	(a) $-\frac{7}{2}$
2. The image of point $(4, -13)$ in line $5x + y + 6 = 0$	(b) $(-1, -14)$
3. Point at equal distance from lines $4x + 3y - 10 - 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$	(c) 90°
4. If slope of line passing through points $(2, 5)$ and $(x, 3)$ is 2 , then the value of x is	(d) $(0, 0)$
5. The slope of line passing through points $(3, -5)$ and $(1, 2)$	(e) 1

Ans. : 1.(c), 2.(b), 3.(d), 4.(e), 5.(a)

2.

Part (A)	Part (B)
1. The slope of line passing through points $(3, -5)$ and $(1, 2)$	(a) $x = 2$
2. Equation of line parallel to x-axis and passing through point $(3, -5)$	(b) $-\frac{7}{2}$
3. Equation of line parallel to x-axis and is at equal distance from lines $x = -2$ and $x = 6$	(c) $y = 2x + 3$
4. Equation of line having slope 2 and which cuts y-intercept as 3 .	(d) $y = -5$
5. Equation of line passing through point $(6, 2)$ having slope -3	(e) $3x + y - 20 = 0$

Ans. : 1.(b), 2.(d), 3.(a), 4.(c), 5.(e)

*** Choose the right answer from the given options. [1 Marks Each]** **[78]**

3. The distance between the orthocentre and circumcentre of the triangle with vertices (1, 2), (2, 1) and $\left(\frac{3+\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$ is:

(A) 0 (B) $\sqrt{2}$ (C) $3 + \sqrt{3}$ (D) none of these.

Ans. :

c. 0

Solution:

Let A(1, 2), B(2, 1) and C $\left(\frac{3+\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$ be the given points.

$\therefore\ \mathrm{AB} = \sqrt{(2-1)^2 + (1-2)^2}$

$= \sqrt{2}$

$\mathrm{BC} = \sqrt{\left(\frac{3+\sqrt{3}}{2} - 2\right)^2, \left(\frac{3+\sqrt{3}}{2} - 1\right)^2}$

$= \sqrt{2}$

$\mathrm{AC} = \sqrt{\left(\frac{3+\sqrt{3}}{2} - 1\right)^2, \left(\frac{3+\sqrt{3}}{2} - 2\right)^2}$

$= \sqrt{2}$

Thus, ABC is an equilateral triangle.

We know that the orthocentre and the circumcentre of an equilateral triangle are same.

So, the distance between the the orthocentre and the circumcentre of the triangle with vertices (1, 2), (2, 1) and $\left(\frac{3+\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$ is 0.

4. Find slope of line joining (1, 2) and (4, 11):

(A) $\frac{1}{3}$ (B) 3 (C) 9 (D) $\frac{1}{9}$

Ans. :

b. 3

Solution:

We know, slope of line joining two points (x_1, y_1) and (x_2, y_2) is given by $\frac{y_2 - y_1}{x_2 - x_1}$

So, slope of line joining (1, 2) and (4, 11) is $\frac{11-2}{4-1} = \frac{9}{3} = 3$

5. The reflection of the point (4, -13) about the line 5x + y + 6 = 0 is:

(A) (-1, -14) (B) (3, 4) (C) (0, 0) (D) (1, 2)

Ans. :

a. (-1, -14)

Solution:

Let the reflection point be A(h, k)

Now, the mid point of line joining (h, k) and (4, -13) will lie on the line 5x + y + 6 = 0

$\therefore 5\left(\frac{h+4}{2}\right) + \frac{k-13}{2} + 6 = 0$

$\Rightarrow 5h + 20 + k - 13 + 12 = 0$

$\Rightarrow 5h + k + 19 = 0 \ldots (1)$

Now, the slope of the line joining points (h, k) and (4, -13) are perpendicular to the line 5x + y + 6 = 0.

slope of the line = -5

slope of line joining by points (h, k) and (4, -13)

$\frac{k+13}{h-4}$

$\therefore \frac{k+13}{h-4}(-5) = -1$

$\Rightarrow 5k - h + 60 = 0 \ldots (2)$

Solving (1) and (2), we get

h = -1 and k = -14

6. The equation of the line passing through (1, 5) and perpendicular to the line 3x - 5y + 7 = 0 is:

(A) 5x + 3y - 20 = 0 (B) 3x - 5y + 7 = 0 (C) 3x - 5y + 6 = 0 (D) 5x + 3y + 7 = 0

Ans. :

a. 5x + 3y - 20 = 0

Solution:

A line perpendicular to 3x - 5y + 7 = 0 is given by

$5x + 3y + \lambda = 0$

This line passes through (1, 5)

$5 + 15 + \lambda = 0$

$\Rightarrow \lambda = -20$

Therefore, the equation of the required line is 5x + 3y - 20 = 0.

7. The line segment joining the points (1, 2) and (-2, 1) is divided by the line 3x + 4y = 7 in the ratio:

(A) 3 : 4 (B) 4 : 3 (C) 9 : 4 (D) 4 : 9

Ans. :

d. 4 : 9

Solution:

Let the line segment joining the points (1, 2) and (−2, 1) be divided by the line 3x + 4y = 7 in the ratio m:n.

Then, the coordinates of this point will be $\left(\frac{-2m+n}{m+n}, \frac{m+2n}{m+n}\right)$ that lie on the line.

$3x + 4y = 7$

$3 \times \frac{-2m+n}{m+n} + 4 \times \frac{m+2n}{m+n} = 7$

$\Rightarrow -2m + 11n = 7m + 7n$

$\Rightarrow -9m = -4n$

$\Rightarrow m:n = 4:9$

8. If p_1 and p_2 are the lengths of the perpendiculars from the origin upon the lines $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then:

(A) $4p_1^2 + p_2^2 = a^2$ (B) $p_1^2 + 4p_2^2 = a^2$ (C) $p_1^2 + 4p_2^2 = a^2$ (D) None of these.

Ans. :

a. $4p_1^2 + p_2^2 = a^2$

Solution:

The given lines are

$x\sec\theta + y\operatorname{cosec}\theta = a \ldots (1)$

$x\cos\theta - y\sin\theta = a\cos 2\theta \ldots (2)$

p_1 and p_2 are the perpendiculars from the origin upon the lines (1) and (2), respectively.

$p_1 = \left|\frac{-a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}\right|$ and $p_2 = \left|\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$\Rightarrow p_1 = \left|\frac{-a\sin\theta\cos\theta}{\sqrt{\sin^2\theta + \cos^2\theta}}\right|$ and $p_2 = \left|-a\cos 2\theta\right|$

$\Rightarrow p_1 = \frac{1}{2}\left|-a \times 2\sin\theta\cos\theta\right|$ and $p_2 = \left|-a\cos 2\theta\right|$

$\Rightarrow p_1 = \frac{1}{2}\left|-a \times \sin 2\theta\right|$ and $p_2 = \left|-a\cos 2\theta\right|$

$\Rightarrow 4p_1^2 + p_2^2 = a^2(\sin^2 2\theta + \cos^2 2\theta)$

$= a^2$

9. Choose the correct answer.

The coordinates of the foot of perpendiculars from the point (2, 3) on the line y = 3x + 4 is given by

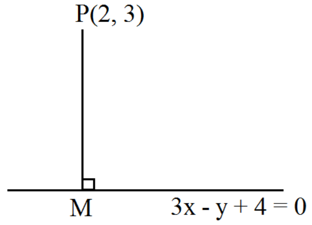
(A) $\frac{37}{10}, \frac{-1}{10}$ (B) $\frac{-1}{10}, \frac{37}{10}$ (C) $\frac{10}{37}, -10$ (D) $\frac{2}{3}, -\frac{1}{3}$

Ans. :

b. $\frac{-1}{10}, \frac{37}{10}$

Solution:

Let the foot of perpendicular from the point P(2, 3) on the line 3x - y + 4 = 0 be M(h, k).



M(h, k) lies on the given line,

$\therefore$ 3h - k + 4 = 0 .....(i)

Also, slopw of the given line is 3.

$\therefore$ Slope of $\mathrm{PM} = -\frac{1}{3} = \frac{\mathrm{k}-3}{\mathrm{h}-2}$ or h + 3k - 11 = 0 .....(ii)

Solving (1) and (ii), we get $(\mathrm{h}, \mathrm{k}) \equiv \left(-\frac{1}{10}, \frac{37}{10}\right)$

10. Area of the triangle formed by the points ((a + 3)(a + 4), a + 3), ((a + 2)(a + 3), (a + 2)) and ((a + 1)(a + 2), (a + 1)) is:

(A) $25a^2$ (B) $5a^2$ (C) $24a^2$ (D) None of these.

Ans. :

d. None of these.

Solution:

The given points are {(a + 3)(a + 4), a + 3), ((a + 2)(a + 3), (a + 2)) and ((a + 1)(a + 2), (a + 1)}

Let *A* be the area of the triangle formed by these points.

Then, $\mathrm{A} = \frac{1}{2}[\mathrm{x}_1(\mathrm{y}_2 - \mathrm{y}_3) + \mathrm{x}_2(\mathrm{y}_3 - \mathrm{y}_1) + \mathrm{x}_3(\mathrm{y}_1 - \mathrm{y}_2)]$

$\Rightarrow \mathrm{A} = \frac{1}{2}[(\mathrm{a}+3)(\mathrm{a}+4)(\mathrm{a}+2-\mathrm{a}-1) + (\mathrm{a}+2)(\mathrm{a}+3)(\mathrm{a}+1-\mathrm{a}-3) + (\mathrm{a}+1)(\mathrm{a}+2)(\mathrm{a}+3-\mathrm{a}-2)]$

$\Rightarrow \mathrm{A} = \frac{1}{2}[(\mathrm{a}+3)(\mathrm{a}+4) - 2(\mathrm{a}+2)(\mathrm{a}+3) + (\mathrm{a}+1)(\mathrm{a}+2)]$

$\Rightarrow \mathrm{A} = \frac{1}{2}[\mathrm{a}^2 + 7\mathrm{a} + 12 - 2\mathrm{a}^2 - 10\mathrm{a} - 12 + \mathrm{a}^2 + 3\mathrm{a} + 2]$

$\Rightarrow \mathrm{A} = 1$

11. The centroid of the triangle with vertices (2, 6), (-5, 6) and (9,3) is:

(A) (2, -3) (B) (2, 5) (C) (-2, 3) (D) (-2, -3)

Ans. :

b. (2, 5)

Solution:

$\mathbf{G} = \left(\frac{\mathbf{x_1+x_2+x_3}}{\mathbf{3}}, \frac{\mathbf{y_1+y_2+y_3}}{\mathbf{3}}\right)$

$= \left(\frac{\mathbf{2-5+9}}{\mathbf{3}}, \frac{\mathbf{6+6+6}}{\mathbf{3}}\right) = \mathbf{(2.5)}$

12. A(6, 3), B(-3, 5), C(4, -2) and D(x, 3x) are four points. If $\triangle \mathrm{DBC} : \triangle \mathrm{ABC} = 1 : 2$, then x is equal to:

(A) $\frac{11}{8}$ (B) $\frac{8}{11}$ (C) 3 (D) None of these

Ans. :

a. $\frac{11}{8}$

Solution:

The area of a triangle with vertices D(x, 3x), B(-3, 5) and C(4, -2) is given below:

Area of $\triangle DBC = \frac{1}{2}\{x(5+2) - 3(-2-3x) + 4(3x-5)\}$

$\Rightarrow$ Area of $\triangle DBC = (14x - 7)$sq units

Similarly, the area of a triangle with vertices A(6, 3), B(-3, 5) and C(4, -2) is given below:

$\triangle ABC = \frac{1}{2}\{6(5+2) - 3(-2-3) + 4(3-5)\}$

$\triangle ABC = \frac{49}{2}$sq units

Given:

$\triangle DBC : \triangle ABC = 1:2$

$\frac{2(14x-7)}{49} = \frac{1}{2}$

$\Rightarrow 8x - 4 = 7$

$\Rightarrow x = \frac{11}{8}$

13. The ratio in which the line 3x + 4y + 2 = 0 divides the distance between the line 3x + 4y + 5 = 0 and 3x + 4y - 5 = 0 is:

(A) 1 : 2 (B) 3 : 7 (C) 2 : 3 (D) 2 : 5

Ans. :

b. 3 : 7

Solution:

Here, in all equations the coefficient of x is same.

It means all the lines have same slope

So, all the lines are parallel.

Now, the distance between the line 3x + 4y + 2 = 0 and 3x + 4y + 5 = 0 is given by

$\frac{|2-5|}{\sqrt{3^2+4^2}}$

$= \frac{3}{\sqrt{25}} = \frac{3}{5}$

Hence, the ratio is given by

$\frac{3}{5} : \frac{7}{5}$

$= 3:7$

14. Slope of a line is given by if inclination of line is α:

(A) $\sin\alpha$ (B) $\cos\alpha$ (C) $\tan\alpha$ (D) $\cot\alpha$

Ans. :

c. $\tan\alpha$

Solution:

Slope of a line is given by $\tan\alpha$ if inclination of line is α Slope is denoted by tangent of the inclination angle.

15. If equation of line is y = 5x + 10 then find the value of x-intercept made by the line:

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{-1}{2}$ (D) -2

Ans. :

d. -2

Solution:

Given, equation is y = 5x + 10. X-intercept means value of x when y is zero. 0 = 5x + 10

$\Rightarrow$ x = -2

16. If p be the length of the perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$, then:

(A) $p^2 = a^2 + b^2$ (B) $p^2 = \frac{1}{a^2} + \frac{1}{b^2}$ (C) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (D) None of these.

Ans. :

c. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution:

It is given that p is the length of the perpendicular from the origin on the line

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{1}{a}x + \frac{1}{b}y - 1 = 0$

$$\therefore p = \left| \frac{0+0+1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right|$$

Squaring both sides,

$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

17. Angle made by line with measured anticlockwise is called inclination of the line:

(A) Positive x-axis (B) Negative x-axis (C) Positive y-axis (D) Negative y-axis

Ans. :

a. Positive x-axis

Solution:

We know, inclination of line is always measured with positive x-axis in anticlockwise direction.

18. The equation of the straight line which passes through the point (-4, 3) such that the portion of the line between the axes is divided internally by the point in the ratio 5 : 3 is:

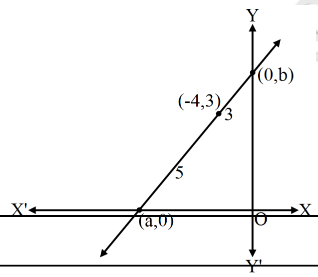
(A) 9x - 20y + 96 = 0 (B) 9x + 20y = 24 (C) 20x + 9y + 53 = 0 (D) none of these.

Ans. :

a. 9x - 20y + 96 = 0

Solution:

Let the required line intersects the coordinate axis at (*a*, 0) and (0, *b*).



The point (−4, 3) divides the required line in the ratio 5 : 3

$\therefore -4 = \frac{5 \times 0 + 3 \times a}{5+3}$ and $3 = \frac{5 \times b + 3 \times 0}{5+3}$

$\Rightarrow a = \frac{-32}{3}$ and $b = \frac{24}{5}$

Hence, The equation of the required line is given below:

$$\frac{x}{\frac{-32}{3}} + \frac{y}{\frac{24}{5}} = 1$$
$$\Rightarrow \frac{-3x}{32} + \frac{5y}{24} = 1$$
$$\Rightarrow -9x + 20y = 96$$
$$\Rightarrow 9x - 20y + 96 = 0$$

19. If -40°F is equal to -40°C and 0°C is equal to 32°F then find the value of 40°C:

(A) 104°F (B) 112°F (C) 86°F (D) 92°F

Ans. :

a. 104°F

Solution:

Let general equation be F = m × c + k

-40 = -40m + k

and 32 = 0 + k

⇒ -40 = -40m + 32

$$m = \frac{72}{40} = \frac{18}{10}$$

$$F = \frac{18}{10} \times 40 + 32$$

= 72 + 32 = 104.

20. Find slope of line if inclination made by the line is 60°.

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) 1

Ans. :

c. $\sqrt{3}$

Solution:

Slope of a line is given by $\tan\alpha$ if inclination of line is α If inclination is 60° the slope is $\tan 60^{\circ} = \sqrt{3}$

21. Find the equation of line parallel to 4x + y = 2 and pass through (2, 5):

(A) 4x + y - 13 = 0 (B) 4x + y + 13 = 0 (C) 4x - y - 13 = 0 (D) 4x - y + 13 = 0

Ans. :

a. 4x + y - 13 = 0

Solution:

Line 4x + y = 2 has slope -4. Line parallel to it has slope -4 and pass through (2, 5)

so equation will be y - 5 = (-4) (x - 2)

⇒ 4x + y - 13 = 0

22. The locus of a point, whose abscissa and ordinate are always equal is:

(A) x - y = 0 (B) x + y = 1 (C) x + y + 1 = 0 (D) None of the above

Ans. :

a. x - y = 0

Solution:

Let the abscissa and ordinate of a point "P" be (x, y)

Given condition: Abscissa = Ordinate

(i.e) x = y

The locus of a point is x - y = 0.

23. The equation of a line that passes through the points (1, 5) and (2, 3) is:

(A) 2x + y - 7 = 0 (B) 2x - y - 7 = 0 (C) x + 2y - 7 = 0 (D) 2x + y + 7 = 0

Ans. :

a. 2x + y - 7 = 0

Solution:

We know that the equation of a line passes through two points (x_1, y_1) and (x_2, y_2) is

$$\frac{(y-y_1)}{(x-x_1)}=\frac{(y_2-y_1)}{(x_2-x_1)}$$

(x_1, y_1) = (1, 5)

(x_2, y_2) = (2, 3)

Now, substitute the values in the formula, we get

$$\frac{(y-5)}{(x-1)}=\frac{(3-5)}{(2-1)}$$

$$\frac{(y-5)}{(x-1)}=\frac{(-2)}{(1)}$$

y - 5 = -2(x - 1)

y - 5 = -2x + 2

2x + y - 5 - 2 = 0

2x + y - 7 = 0

∴ The equation of a line that passes through the points (1, 5) and (2, 3) is 2x + y - 7 = 0.

24. If a line with slope m makes x-intercept d. Then equation of the line is:

(A) y = m(d - x) (B) y = m(x - d) (C) y = m(x + d) (D) y = mx + d

Ans. :

b. y = m(x - d)

25. If the area of the triangle with vertices (x, 0), (1, 1) and (0, 2) is 4 square unit, then the value of x is:

(A) -2 (B) -4 (C) -6 (D) 8

Ans. :

c. -6

26. If the two lines are perpendicular then difference of their inclination angle is:

(A) 45° (B) 60° (C) 90° (D) 180°

Ans. :

c. 90°

Solution:

If the two lines are perpendicular then if one line form angle α with positive x-axis then the other line form angle $90^\circ+\alpha$

27. The medians AD and BE of a triangle with vertices A(0, b), B(0, 0) and C(a, 0) are perpendicular to each other, if

(A) $a=\frac{b}{2}$ (B) $b=\frac{a}{2}$ (C) $ab=1$ (D) $a=\pm\sqrt{2}b$

Ans. :

d. $a=\pm\sqrt{2}b$

Solution:

The midpoints of BC and AC are $D\left(\frac{a}{2},0\right)$ and $E\left(\frac{a}{2},\frac{b}{2}\right)$

Slope of $AD=\frac{0-b}{\frac{a}{2}-0}$

Slope of BE $= \frac{-\frac{b}{2}}{\frac{-a}{2}}$

It is given that the medians are perpendicular to each other.

$\frac{0-b}{\frac{a}{2}-0} \times \frac{-\frac{b}{2}}{-\frac{a}{2}} = -1$

$\Rightarrow a = \pm\sqrt{2}b$

28. In what ratio does the line y - x + 2 = 0 cut the line joining (3, -1) and (8, 9)?

(A) 2 : 3 (B) 3 : 2 (C) 3 : -2 (D) 1 : 2

Ans. :

a. 2 : 3

29. Slope of a line is given by if inclination of line is α:

(A) $\sin\alpha$ (B) $\cos\alpha$ (C) $\tan\alpha$ (D) $\cot\alpha$

Ans. :

c. $\tan\alpha$

Solution:

Slope of a line is given by $\tan\alpha$ if inclination of line is α. Slope is denoted by tangent of the inclination angle.

30. If (-4, 5) is one vertex and 7x - y + 8 = 0 is onediagonal of a square, then the equation of second diagonal is:

(A) x + 3y = 21 (B) 2x - 3y = 7 (C) x + 7y = 31 (D) 2x + 3y = 21

Ans. :

c. x + 7y = 31

31. A line passes through (2, 2) and is perpendicular to the line 3x + y = 3. Its y-intercept is:

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{4}{3}$

Ans. :

d. $\frac{4}{3}$

32. If P (1, 2), Q (3, 5), R (7, 9) form a triangle then find the equation of median through P:

(A) 5x - 4y + 3 = 0 (B) 5x + 4y + 3 = 0 (C) 5x - 4y - 3 = 0 (D) 5x + 4y - 3 = 0

Ans. :

a. 5x - 4y + 3 = 0

Solution:

Midpoint of QR line is $\left(\frac{3+7}{2}, \frac{5+9}{2}\right) = (5, 7)$

Equation of line joining (1, 2) and (5, 7) is $\frac{y-2}{7-2} = \frac{x-1}{5-1}$

$\Rightarrow \frac{y-2}{5} = \frac{x-1}{4}$

⇒ 4y - 8 = 5x - 5

⇒ 5x - 4y + 3 = 0.

33. If the two lines with slope m_1 and m_2 are perpendicular then their slopes has relation:

(A) $m_1 + m_2 = 1$ (B) $m_1 \times m_2 = 1$ (C) $m_1 \times m_2 = -1$ (D) $m_1 + m_2 = -1$

Ans. :

c. $m_1 \times m_2 = -1$

Solution:

If the two lines are perpendicular then if one line form angle α with positive x-axis then the other line form angle $90^\circ + \alpha$

If $m_1 = \tan\alpha$ then m_2 will be $\tan(90^\circ + \alpha) = -\cot\alpha = \frac{-1}{\tan\alpha}$

$\Rightarrow m_1 \times m_2 = -1$.

34. If the points A (1, 2), B (2, 4) and C (3, a) are collinear, what is the length BC?

(A) 2 unit (B) 3 unit (C) 5 unit (D) 5 unit

Ans. :

c. 5 unit

35. Find slope of line passing through origin and (3, 6):

(A) 2 (B) 3 (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Ans. :

a. 2

Solution:

We know, slope of line joining two points (x_1, y_1) and (x_2, y_2) is given by $\frac{(y_2 - y_1)}{(x_2 - x_1)}$

So, slope of line joining (0, 0) and (3, 6) is $\frac{(6-0)}{(3-0)} = \frac{6}{3} = 2$

36. Choose the correct answer.

The tangent of angle between the lines whose intercepts on the axes are a, -b and b, -a, respectively, is

(A) $\frac{a^2 - b^2}{ab}$ (B) $\frac{b^2 - a^2}{2}$ (C) $\frac{b^2 - a^2}{2ab}$ (D) None of these.

Ans. :

c. $\frac{b^2 - a^2}{2ab}$

Solution:

Intercepts of line are a and -b; i.e., line passes through the points (a, 0), (0, -b).

$\therefore$ Slope of line, $m_1 = \frac{-b-0}{0-a} = \frac{b}{a}$

Intercepts of line are b, -a; i.e., line passes through the points (b, 0), (0, -a).

$\therefore$ Slope of line, $m_2 = \frac{-a-0}{0-b} = \frac{a}{b}$

If θ is the angle between the lines, then

$$\tan = \theta = \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{a}{b} \times \frac{b}{a}} = \frac{\frac{b^2 - a^2}{ab}}{2} = \frac{b^2 - a^2}{2ab}$$

37. If slope of a line is $\frac{2}{3}$ then find the slope of line perpendicular to it:

(A) $\frac{-3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

Ans. :

a. $\frac{-3}{2}$

Solution:

If lines with slopes m_1 and m_2 are perpendicular then $m_1 \times m_2 = -1$.

If $m_1 = \frac{2}{3}$ then $m_2 = \frac{-3}{2}$

38. Three vertices of a parallelogram taken in order are (-1, -6), (2, -5) and (7, 2). The fourth vertex is:

(A) (1, 4) (B) (4, 1) (C) (1, 1) (D) (4, 4)

Ans. :

b. (4, 1)

Solution:

Let A(-1, -6), B(2, -5) and C(7, 2) be the given vertex. Let D(h, k) be the fourth vertex.

The midpoints of AC and BD are (3, -2) and $\left(\frac{2+h}{2}, \frac{-5+k}{2}\right)$ respectively.

We know that the diagonals of a parallelogram bisect each other.

$\therefore 3 = \frac{2+h}{2}$ and $-2 = \frac{-5+k}{2}$

$\Rightarrow h = 4$ and $k = 1$

39. Find the equation of line parallel to 4x + y = 2 and pass through (2, 5):

(A) 4x + y - 13 = 0 (B) 4x + y + 13 = 0 (C) 4x - y - 13 = 0 (D) 4x - y + 13 = 0

Ans. :

a. 4x + y - 13 = 0

Solution:

Line 4x + y = 2 has
slope -4 Line parallel to it has
slope -4 and pass through (2, 5)
so equation will be y - 5 = (-4) (x - 2)
$\Rightarrow$ 4x + y - 13 = 0

40. Two lines are said to be perpendicular if the product of their slope is equal to:

(A) -1 (B) 0 (C) 1 (D) $\frac{1}{2}$

Ans. :

a. -1

Solution:

When two lines are perpendicular, then the product of their slope is equal to -1. If two lines are perpendicular with slope m_1 and m_2, then $m_1.m_2 = -1$.

41. L is a variable line such that the algebraic sum of the distances of the points (1, 1), (2, 0) and (0, 2) from the line is equal to zero. The line L will always pass through:

a. (1, 1)
b. (2, 1)
c. (1, 2)
d. none of these.

Ans. :

a. (1,1)

Solution:

Let ax + by + c = 0 be the variable line. It is given that the algebraic sum of the distances of the points (1, 1), (2, 0) and (0, 2) from the line is equal to zero

$$\therefore \frac{a+b+c}{\sqrt{a^2+b^2}} + \frac{2a+0+c}{\sqrt{a^2+b^2}} + \frac{0+2b+c}{\sqrt{a^2+b^2}} = 0$$

$\Rightarrow 3a + 3b + 3c = 0$

$\Rightarrow a + b + c = 0$

Substituting c = -a - b in ax + by + c = 0, we get:

$ax + by - a - b = 0$

$\Rightarrow a(x - 1) + b(y - 1) = 0$

$\Rightarrow (x - 1) + \frac{a}{b}(y - 1) = 0$

This line is of the form $L_1 + \lambda L_2 = 0$ which passes through the intersection of L_1 =0 and L_2 =0,i.e. x - 1 = 0 and y - 1 = 0.

$\Rightarrow$ x = 1, y = 1

42. The equation of line passing through origin $(0, 0)$ and point $(a\cos\theta, a\sin\theta)$ is :

(A) $y = x\cos\theta$ (B) $y = x\tan\theta$ (C) $y = x\sin\theta$ (D) $y = x\cot\theta$

Ans. : (B) $y = x\tan\theta$

43. Equation of line parallel to $3x - 4y = 7$ and passing through origin $(0, 0)$ is given by :

(A) $3x - 4y = 1$ (B) $3x - 4y = 0$

(C) $4x - 3y = 1$ (D) $3y - 4x = 0$

Ans. : (B) $3x - 4y = 0$

44. The length of perpendicular drawn from origin on line $x + \sqrt{3}y = 1$ is p. The value of p is:

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1

Ans. : (B) $\frac{1}{2}$

45. If lines $y = mx + 5$ and $3x + 5y = 8$ are mutually perpendicular then the value of m is :

(A) $\frac{5}{3}$ (B) $-\frac{5}{3}$ (C) $-\frac{3}{5}$ (D) $\frac{3}{5}$

Ans.: (A) $\frac{5}{3}$

46. Equation of line perpendicular to straight line $3x - 4y + 7 = 0$ and passing through point $(1, -2)$ is given by :

(A) $4x + 3y - 2 = 0$ (B) $4x + 3y + 2 = 0$

(C) $4x - 3y + 2 = 0$ (D) $4x - 3y - 2 = 0$

Ans. : (B) $4x + 3y + 2 = 0$

47. The distance between lines $4x - 3y + 8 = 0$ and $3y - 4x - 6 = 0$ is given by :

(A) 14 (B) 2 (C) $\frac{14}{5}$ (D) $\frac{2}{5}$

Ans. : (D) $\frac{2}{5}$

48. Point at lines $3y + x - 10 = 0$ and $2x - 5y + 13 = 0$ is:

(A) $(0, 0)$ (B) $(6, 5)$ (C) $(4, 2)$ (D) $(1, 3)$

Ans. : (D) $(1, 3)$

49. If lines $y = mx + c$ and $x = my + c$ are mutually perpendicular then the value of m is :

(A) 1 (B) -1

(C) 0 (D) Cannot be determined

Ans. : (B) -1

50. Line $x = 3, y = 4$ and $4x - 3y + a = 0$ are coincidence, then the value of a is :

(A) 12 (B) -12 (C) 0 (D) -7

Ans. : (C) 0

51. If lines $a_1x - b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are mutually perpendicular then :

(A) $a_1b_2 + a_2b_1 = 0$ (B) $a_1a_2 + b_1b_2 = 0$

(C) $a_1b_2 - a_2b_1 = 0$ (D) $a_1a_2 - b_1b_2 = 0$

Ans. : (D) $a_1a_2 - b_1b_2 = 0$

52. The image of point $(3, 8)$ in line $x + 2y - 7 = 0$ is given by:

(A) $(-1, -4)$ (B) $(-3, -8)$ (C) $(1, -4)$ (D) $(3, 8)$

Ans. : (B) $(-3, -8)$

53. If line passing through point $(4, 3)$ and $(2, k)$ is perpendicular to the line $y = 2x + 3$, then k equals to :

(A) 2 (B) 3 (C) 4 (D) 5

Ans. : (C) 4

54. A line L passes through the points $(1, 1)$ and $(2, 0)$ and another line L' passes through $\left(\frac{1}{2}, 0\right)$ and perpendicular to L. Then the area of the triangle formed by the lines L, L' and y- axis, is

(A) $\frac{15}{8}$ (B) $\frac{25}{4}$ (C) $\frac{25}{8}$ (D) $\frac{25}{16}$

Ans. : d

(d) Here $L \equiv x + y = 2$and $L' \equiv 2x - 2y = 1$.

Equation of y-axis is $x = 0$

Hence the vertices of the triangle are $A(0, 2), B\left(0, -\frac{1}{2}\right)$ and $C\left(\frac{5}{4}, \frac{3}{4}\right)$.

Therefore, the area of the triangle is $\frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 0 & -\frac{1}{2} & 1 \\ \frac{5}{4} & \frac{3}{4} & 1 \end{vmatrix} = \frac{25}{16}$.

55. The sides AB, BC, CD and DA of a quadrilateral are $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively. The angle between diagonals AC and BD is ......^{o}

(A) 45 (B) 60 (C) 90 (D) 30

Ans. : c

(c) The four vertices on solving are $A(-3, 3)$, $B(1, 1)$, $C(1, -1)$ and $D(-2, -2)$.m_1= slope of $AC = -1$, m_2= slope of $BD = 1$; $m_1m_2 = -1$.

Hence the angle between diagonals AC and BD is 90°.

56. If A is $(2, 5)$, B is $(4, -11)$ and C lies on $9x + 7y + 4 = 0$, then the locus of the centroid of the ΔABC is a straight line parallel to the straight line is

(A) $7x - 9y + 4 = 0$ (B) $9x - 7y - 4 = 0$ (C) $9x + 7y + 4 = 0$ (D) $7 + 9y + 4 = 0$

Ans. : c

(c) According to question , $x_1 = \frac{2+4+x}{3} \Rightarrow x = 3x_1 - 6$

$y_1 = \frac{5-11+y}{3} \Rightarrow y = 3y_1 + 6$

$\therefore\ 9(3x_1 - 6) + 7(3y_1 + 6) + 4 = 0$

Hence locus is $27x + 21y - 8 = 0$, which is parallel to $9x + 7y + 4 = 0$.

57. A point moves so that square of its distance from the point $(3, -2)$ is numerically equal to its distance from the line $5x - 12y = 13$. The equation of the locus of the point is

(A) $13x^2 + 13y^2 - 83x + 64y + 182 = 0$

(B) $x^2 + y^2 - 11x + 16y + 26 = 0$

(C) $x^2 + y^2 - 11x + 16y = 0$

(D) None of these

Ans. : a

(a) $(h-3)^2 + (k+2)^2 = \left|\frac{5h-12k-13}{\sqrt{25+144}}\right|$.

Replace (h, k) by $(x,\ y)$, we get

$13x^2 + 13y^2 - 83x + 64y + 182 = 0$, which is the required equation of the locus of the point.

58. The equation to the sides of a triangle are $x - 3y = 0$, $4x + 3y = 5$ and $3x + y = 0$. The line $3x - 4y = 0$ passes through

(A) The incentre

(B) The centroid

(C) The circumcentre

(D) The orthocentre of the triangle

Ans. : d

(d) Two sides $x - 3y = 0$and $3x + y = 0$of the given triangle are perpendicular to each other. Therefore its orthocentre is the point of intersection of $x - 3y = 0$ and $3x + y = 0$ i.e, $(0, 0)$. Clearly satisfies $3x - 4y = 0$.

59. The vertex of an equilateral triangle is $(2, -1)$ and the equation of its base in $x + 2y = 1$. The length of its sides is

(A) $4/\sqrt{15}$ (B) $2/\sqrt{15}$ (C) $4/3\sqrt{3}$ (D) $1/\sqrt{5}$

Ans. : b

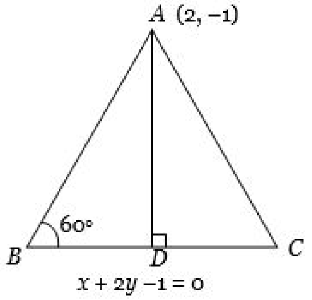
(b) $|AD| = \left|\frac{2-2-1}{\sqrt{1^2+2^2}}\right| = \frac{1}{\sqrt{5}}$

$\tan 60^\circ = \frac{AD}{BD}$

==> $\sqrt{3} = \frac{1/\sqrt{5}}{BD}$

==> $BD = \frac{1}{\sqrt{15}}$

$BC = 2BD = 2/\sqrt{15}$.



60. $A(-1, 1)$, $B(5, 3)$ are opposite vertices of a square in xy-plane. The equation of the other diagonal (not passing through (A, B) of the square is given by

(A) $x - 3y + 4 = 0$ (B) $2x - y + 3 = 0$ (C) $y + 3x - 8 = 0$ (D) $x + 2y - 1 = 0$

Ans. : c

(c) The required diagonal passes through the mid-point of AB and is perpendicular to AB. So its equation is $y - 2 = -3(x - 2)$ or $3x + y - 8 = 0$.

61. The opposite angular points of a square are $(3,\ 4)$ and $(1,\ -1)$. Then the co-ordinates of other two points are

(A) $D\left(\frac{1}{2},\ \frac{9}{2}\right),\ B\left(-\frac{1}{2},\ \frac{5}{2}\right)$ (B) $D\left(\frac{1}{2},\ \frac{9}{2}\right),\ B\left(\frac{1}{2},\ \frac{5}{2}\right)$

(C) $D\left(\frac{9}{2},\ \frac{1}{2}\right),\ B\left(-\frac{1}{2},\ \frac{5}{2}\right)$ (D) None of these

Ans. : c

(c) Obviously, slope of $AC = 5/2$.

Let m be the slope of a line inclined at an angle of 45^o to AC, then

$\tan 45^o = \pm \frac{m - \frac{5}{2}}{1 + m.\frac{5}{2}} \Rightarrow m = -\frac{7}{3}, \frac{3}{7}$.

Thus, let the slope of AB or DC be $3/7$ and that of AD or BC be $-\frac{7}{3}$. Then equation of AB is $3x - 7y + 19 = 0$.

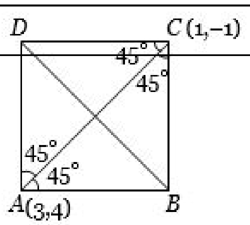
Also the equation of BC is $7x + 3y - 4 = 0$.

On solving these equations, we get, $B\ \left(-\frac{1}{2}, \frac{5}{2}\right)$.

Now let the coordinates of the vertex D be (h, k). Since the middle points of AC and BD are same,

therefore $\frac{1}{2}\left(h - \frac{1}{2}\right) = \frac{1}{2}(3 + 1) \Rightarrow h = \frac{9}{2}, \frac{1}{2}\left(k + \frac{5}{2}\right) = \frac{1}{2}(4 - 1)$

==> $k = \frac{1}{2}$. Hence, $D = \left(\frac{9}{2}, \frac{1}{2}\right)$.



62. If the coordinates of the points A, B, C be $(-1, 5)$, $(0, 0)$ and $(2, 2)$ respectively and D be the middle point of BC, then the equation of the perpendicular drawn from B to the line AD is

(A) $x + 2y = 0$ (B) $2x + y = 0$ (C) $x - 2y = 0$ (D) $2x - y = 0$

Ans. : c

(c) Here $D(1, 1)$ therefore equation of line AD is given by $2x + y - 3 = 0$. Thus the line perpendicular to AD is $x - 2y + k = 0$and it passes through B, so $k = 0$. Hence required equation is $x - 2y = 0$.

63. A straight line passes through a fixed point (h, k). The locus of the foot of perpendicular on it drawn from the origin is

(A) $x^2 + y^2 - hx - ky = 0$ (B) $x^2 + y^2 + hx + ky = 0$

(C) $3x^2 + 3y^2 + hx - ky = 0$ (D) None of these

Ans. : a

(a) $y - k = m(x - h)$ and $y - 0 = -\frac{1}{m}(x - 0)$. Eliminate m and replace (h, k) by $(x,\ y)$, we get

$x^2 + y^2 - hx - ky = 0$, which is the required locus of the point.

64. If $(-2, 6)$ is the image of the point $(4, 2)$ with respect to line $L = 0$, then $L =$

(A) $3x - 2y + 5$ (B) $3x - 2y + 10$ (C) $2x + 3y - 5$ (D) $6x - 4y - 7$

Ans. : a

(a) The mid point of $P(-2,\ 6)$ and $Q(4,\ 2)$ is $\left(\frac{-2+4}{2}, \frac{6+2}{2}\right)$ i.e., $(1,\ 4)$ and the gradient of line

$PQ = \frac{2-6}{4+2} = \frac{-2}{3}$

The slope of $L = \frac{3}{2}$

Hence the equation of line which passes through point $(1,\ 4)$ is $y - 4 = \frac{3}{2}(x - 1)$
==> $3x - 2y + 5 = 0$.

65. The image of a point $A(3,\ 8)$ in the line $x + 3y - 7 = 0$, is

(A) $(-1,\ -4)$ (B) $(-3\ ,\ -8)$ (C) $(1,\ -4)$ (D) $(3,\ 8)$

Ans. : a

(a) Equation of the line passing through $(3, 8)$ and perpendicular to $x + 3y - 7 = 0$is $3x - y - 1 = 0$. The intersection point of both the lines is $(1, 2)$. Now let the image of $A(3, 8)$ be $A'(x_1, y_1)$, then point $(1, 2)$ will be the mid point of AA'.

==> $\frac{x_1+3}{2} = 1$

$\Rightarrow x_1 = -1$ and $\frac{y_1+8}{2} = 2$

==> $y_1 = -4$.

Hence the image is $(-1,\ -4)$.

66. The coordinates of the foot of the perpendicular from $(x_1, y_1)$to the line $ax + by + c = 0$ are

(A) $\left(\frac{b^2x_1 - aby_1 - ac}{a^2+b^2}, \frac{a^2y_1 - abx_1 - bc}{a^2+b^2}\right)$ (B) $\left(\frac{b^2x_1 - aby_1 + ac}{a^2+b^2}, \frac{a^2y_1 + abx_1 + bc}{a^2+b^2}\right)$ (C) $\left(\frac{ax_1 + by_1 + ab}{a+b}, \frac{ax_1 - by_1 - ab}{a+b}\right)$ (D) None of these

Ans. : a

(a)It is a fundamental concept.

67. The coordinates of the foot of the perpendicular from the point $(2, 3)$ on the line $y = 3x + 4$ are given by

(A) $\left(\frac{37}{10}, -\frac{1}{10}\right)$ (B) $\left(-\frac{1}{10}, \frac{37}{10}\right)$ (C) $\left(\frac{10}{37}, -10\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}\right)$

Ans. : b

(b)$-y + 3x + 4 = 0$and perpendicular is $\frac{y-3}{x-2} = \frac{-1}{3}$ or $3y + x - 11 = 0$. Therefore foot is $x = \frac{-1}{10}, y = \frac{37}{10}$.

68. If the lines $ax + 2y + 1 = 0, bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent, then a, b, c are in

(A) $A. P.$ (B) $G. P$ (C) $H. P.$ (D) None of these

Ans. : a

(a) It is given that the lines $ax + 2y + 1 = 0, bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$

are concurrent, therefore $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$

==> $-a + 2b - c = 0.$

$\Rightarrow 2b = a + c$

==> a, b, c are in $A. P.$

69. If a and b are two arbitrary constants, then the straight line $(a - 2b)x + (a + 3b)y + 3a + 4b = 0$ will pass through

(A) $(-1, -2)$ (B) $(1, 2)$ (C) $(-2, -3)$ (D) $(2, 3)$

Ans. : a

(a) $(a - 2b)x + (a + 3b)y + 3a + 4b = 0$

or $a(x + y + 3) + b(-2x + 3y + 4) = 0$, which represents a family of straight lines through point of intersection of $x + y + 3 = 0$ and $-2x + 3y + 4 = 0$i.e , $(-1, -2)$.

Trick : Point $(-1, -2)$ satisfies the given equation of straight line.

70. Let α be the distance between the lines $-x + y = 2$ and $x - y = 2$, and β be the distance between the lines $4x - 3y = 5$ and $6y - 8x = 1$, then

(A) $20\sqrt{2}\beta = 11\alpha$ (B) $20\sqrt{2}\alpha = 11\beta$ (C) $11\sqrt{2}\beta = 20\alpha$ (D) None of these

Ans. : a

(a) Distance between lines $-x + y = 2$ and $x - y = 2$ is , $\alpha = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$

Distance between lines $4x - 3y = 5$ and $6y - 8x = 1$ is,

$\beta = \frac{1}{10} + \frac{5}{5} = \frac{11}{10}$

Therefore $\frac{\alpha}{\beta} = \frac{2\sqrt{2}}{11/10} \Rightarrow 20\sqrt{2}\beta = 11\alpha.$

71. $(\sin\theta, \cos\theta)$ and $(3, 2)$ lies on the same side of the line $x + y = 1$, then θ lies
between

(A) $(0, \ \pi/2)$ (B) $(0, \ \pi)$ (C) $(\pi/4, \pi/2)$ (D) $(0, \ \pi/4)$

Ans. : d

(d)As $(\sin\theta, \cos\theta)$and $(3, 2)$ lie on the same side of $x + y - 1 = 0$, they should be of same sign.

$\sin\theta + \cos\theta - 1 >$ as $3 + 2 - 1 > 0$

==> $\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) > 1$

==> $\sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$.

72. The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$, is

(A) $7:3$ (B) $3:7$ (C) $2:3$ (D) None of these

Ans. : b

(b)Lines $3x + 4y + 2 = 0$and $3x + 4y + 5 = 0$ are on the same side of the origin.

The distance between these lines is $d_1 = \left|\frac{2-5}{\sqrt{3^2+4^2}}\right| = \frac{3}{5}$.

Lines $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$are on the opposite sides of the origin.

The distance between these lines is $d_2 = \left|\frac{2+5}{\sqrt{3^2+4^2}}\right| = \frac{7}{5}$.

Thus $3x + 4y + 2 = 0$ divides the distance between $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ in the ratio $d_1 : d_2$ i.e., $3:7$.

73. If straight lines $\alpha^2 x + \alpha y = 9$ and $3x + 2y = 5$ are perpendicular, then the value of α is

(A) $-2/3$ (B) 0 (C) $-3/2$ (D) $2/3$

Ans.: (A) $-2/3$

74. The straight line passing through the point of intersection of the straight lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and having infinite slope and at a distance of 2 *units* from the origin, has the equation

(A) $x = 2$ (B) $3x + y - 1 = 0$ (C) $y = 1$ (D) None of these

Ans. : a

(a) The intersection point of $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ is $(2, 1)$ and $m = \frac{1}{0}$. So the required line is $y - 1 = \frac{1}{0}(x - 2) \Rightarrow x = 2$.

75. The point $P(a, \ b)$ lies on the straight line $3x + 2y = 13$ and the point $Q\,(b, \ a)$ lies on the straight line $4x - y = 5$, then the equation of line PQ is

(A) $x - y = 5$ (B) $x + y = 5$ (C) $x + y = -5$ (D) $x - y = -5$

Ans. : b

(b) Point $P(a, b)$ is on $3x + 2y = 13$

So, $3a + 2b = 13$.....(i)

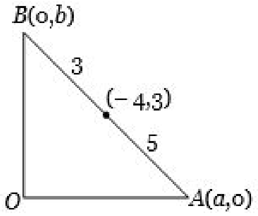
Point $Q(b, a)$is on $4x - y = 5$
So, $4b - a = 5$.....(ii)
By solving (i) and (ii), $a = 3, b = 2$
$P(a, b) \rightarrow (3,\ 2)$and $Q(b, a) \rightarrow (2,\ 3)$
Now, equation of PQ $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$\Rightarrow y - 2 = \frac{3-2}{2-3}(x - 3)$
==> $y - 2 = -(x - 3)$
$\Rightarrow x + y = 5.$

76. Equation of the line which passes through the point $(-4,\ 3)$ and the portion of the line intercepted between the axes is divided internally in the ratio $5:3$ by this point, is

(A)	(B)	(C)	(D) None of these
$9x + 20y + 96 = 0$	$20x + 9y + 96 = 0$	$9x - 20y + 96 = 0$	

Ans. : c

(c) By the section formula, we get $a = -\frac{32}{3}$ and $b = \frac{24}{5}$. Hence the required equation is given by
$\frac{x}{-(32/3)} + \frac{y}{(24/5)} = 1$
$\Rightarrow 9x - 20y + 96 = 0.$



77. The equations of the lines passing through the point $(1, 0)$ and at a distance $\frac{\sqrt{3}}{2}$ from the origin, are
(A) $\sqrt{3}x + y - \sqrt{3} = 0,\ \ \sqrt{3}x - y - \sqrt{3} = 0$
(B) $\sqrt{3}x + y + \sqrt{3} = 0,\ \ \sqrt{3}x - y + \sqrt{3} = 0$
(C) $x + \sqrt{3}y - \sqrt{3} = 0,\ \ x - \sqrt{3}y - \sqrt{3} = 0$
(D) None of these

Ans. : a
(a) The equation of lines passing through $(1, 0)$ are given by $y = m(x - 1)$. Its distance from origin is $\frac{\sqrt{3}}{2}$.

$$\Rightarrow \left| \frac{-m}{\sqrt{1+m^2}} \right| = \frac{\sqrt{3}}{2}$$

$\Rightarrow m = \pm\sqrt{3}.$
Hence the lines are $\sqrt{3}x + y - \sqrt{3} = 0$ and $\sqrt{3}x - y - \sqrt{3} = 0.$

78. A line passes through the point $(3, 4)$ and cuts off intercepts from the coordinates axes such that their sum is 14. The equation of the line is
(A) $4x - 3y = 24$ (B) $4x + 3y = 24$ (C) $3x - 4y = 24$ (D) $3x + 4y = 24$

Ans. : b
(b) Given $a + b = 14 \Rightarrow a = 14 - b$

Hence the equation of straight line is $\frac{x}{14-b} + \frac{y}{b} = 1$.
Also, it passes through $(3, 4)$

$\therefore \frac{3}{14-b} + \frac{4}{b} = 1 \Rightarrow b = 8$ or 7

Therefore equations are $4x + 3y = 24$ and $x + y = 7$.

Trick : This question can be checked with the options as the line $4x + 3y = 24$ passes through $(3, 4)$ and also cuts the intercepts from the axes whose sum is 14.

79. If the transversal $y = m_r x$; $r = 1, 2, 3$ cut off equal intercepts on the transversal $x + y = 1$, then $1 + m_1$, $1 + m_2$, $1 + m_3$ are in

(A) $A. P.$ (B) $G. P.$ (C) $H. P.$ (D) None of these

Ans. : c

(c) Solving $y = m_r x$ and $x + y = 1$, we get $x = \frac{1}{1+m_r}$ and $y = \frac{m_r}{1+m_r}$.

Thus the points of intersection of the three lines on the transversal are

$$\left(\frac{1}{1+m_1}, \frac{m_1}{1+m_1}\right), \left(\frac{1}{1+m_2}, \frac{m_2}{1+m_2}\right) \text{ and } \left(\frac{1}{1+m_3}, \frac{m_3}{1+m_3}\right)$$

By hypothesis, $\left(\frac{1}{1+m_1} - \frac{1}{1+m_2}\right)^2 + \left(\frac{m_1}{1+m_1} - \frac{m_2}{1+m_2}\right)^2$

$= \left(\frac{1}{1+m_2} - \frac{1}{1+m_3}\right)^2 + \left(\frac{m_2}{1+m_2} - \frac{m_3}{1+m_2}\right)^2$

==> $\frac{m_2 - m_1}{1+m_1} = \frac{m_3 - m_2}{1+m_3}$ or $\frac{1+m_2}{1+m_1} - 1 = 1 - \frac{1+m_2}{1+m_3}$

==> $\frac{1+m_2}{1+m_1} + \frac{1+m_2}{1+m_3} = 2$ ==> $1 + m_2 = \frac{2(1+m_1)(1+m_3)}{(1+m_1)+(1+m_3)}$

==> $1 + m_1, 1 + m_2, 1 + m_3$ are in $H. P.$

80. If the coordinates of the points A and B be $(3, 3)$ and $(7, 6)$, then the length of the portion of the line AB intercepted between the axes is

(A) $\frac{5}{4}$ (B) $\frac{\sqrt{10}}{4}$ (C) $\frac{\sqrt{13}}{3}$ (D) None of these

Ans. : a

(a) Equation of line AB is $y - 3 = \frac{6-3}{7-3}(x - 3)$

==> $3x - 4y + 3 = 0 \Rightarrow \frac{x}{-1} + \frac{y}{3/4} = 1$

Hence required length is $\sqrt{(-1)^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4}$.

*** Given section consists of questions of 2 marks each.** **[62]**

81. Find the value of x for which the points (x, -1)(2, 1) and (4, 5) are collinear.

Ans. : Let A(x, -1), B (2, 1) and C (4, 5) be three collinear points.

$\therefore$ Slope of AB $= \frac{1-(-1)}{2-x} = \frac{1+1}{2-x} = \frac{2}{2-x}$

Slope of BC = $\frac{5-1}{4-2} = \frac{4}{2} = 2$

Since points A,Band C are collinear therefore slope of AB=slope of BC

2/2-x=2

Hence x=2

82. Find the equation of the line which satisfy the given conditions:
Passing through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of 75^o.

Ans. : It is given that the point = $(2, 2\sqrt{3})$ and angle $\theta = 75°$

Equation of line: $(y - y_1) = m (x - x_1)$

where, m = slope of line = tan θand (x_1, y_1) are the points through which line passes

$\therefore$ m = tan 75°

75° = 45° + 30°Applying the formula: tan (A + B) = $\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\tan\left(45^\circ + 30^\circ\right) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Rationalizing we obtain$\tan 75^\circ = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_1, y_1), if and only if, its coordinates satisfy the equation $y - y_1 = m (x - x_1)$

$\therefore\ y - 2\sqrt{3} = (2 + \sqrt{3})(x - 2)$

$\Rightarrow y - 2\sqrt{3} = 2x - 4 + \sqrt{3}x - 2\sqrt{3}$

$\Rightarrow y = 2x - 4 + \sqrt{3}x$

$\Rightarrow (2 + \sqrt{3})x - y - 4 = 0$

Therefore, the equation of the line is $(2 + \sqrt{3})$ x – y - 4 = 0.

83. Find the equation of the line which satisfy the given condition:
The line passing through the points (-1, 1) and (2, - 4).

Ans. : Given points are $A(x_1, y_1) = (-1, 1)$ and $B(x_2, y_2) = (2, -4)$, then equation of line AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}\left(x - x_1\right)$$

$\Rightarrow \quad y - 1 = \frac{-4 - 1}{2 + 1}(x + 1)\left[\because x_1 = -1, y_1 = 1, x_2 = 2, y_2 = -4\right]$

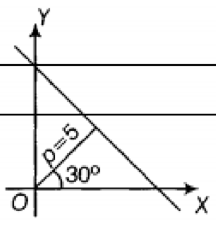
$\Rightarrow \quad y - 1 = \frac{-5}{3}(x + 1) \Rightarrow 3y - 3 = -5x - 5$

$\Rightarrow$ 5x + 3y + 2 = 0

84. Find the equation of the line which satisfy the given condition:
Perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive X-axis is 30^o.

Ans. :

Here, p = 5 and $\alpha = 30^o$



So, equation of line in normal form is

x cosα + y sinα = p

$\therefore$ x cos 30° + y sin 30° = 5

$\Rightarrow \quad \frac{\sqrt{3}}{2}x + \frac{1}{2}y = 5$

$\Rightarrow \quad \sqrt{3}x + y = 10$

85. Find the distance between parallel lines. l(x + y) + p = 0 and l(x + y) - r = 0

Ans. : We have the equation,

lx + ly + p = 0

and lx + ly - r = 0

where a = 1, b = 1 , c_1 = p and c_2 = -r

$\therefore$ The distance between two parallel lines

d = $\frac{\left|c_1 - c_2\right|}{\sqrt{a^2+b^2}}$

$\Rightarrow \frac{|p+r|}{\sqrt{1^2+1^2}}$

$\Rightarrow \frac{1}{\sqrt{2}}\left|\frac{p+r}{1}\right|$ units

86. Find the equation of the line perpendicular to the line x - 7y + 5 = 0 and having x intercept 3.

Ans. : Equation of any line which is perpendicular to the line

x - 7y + 5 = 0 is 7x + y + k = 0

Since this line passes through point (3, 0)

$\therefore 7 \times 3 + 0 + k = 0 \Rightarrow$ k = -21

Thus equation of required line is 7x + y - 21 = 0

87. Find the angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Ans. : We have $\sqrt{3}x + y = 1$

$\Rightarrow y = -\sqrt{3}x + 1$

$\therefore m_1 = -\sqrt{3}$

Also $x + \sqrt{3}y = 1$

$\Rightarrow \sqrt{3}y = -x + 1$

$\Rightarrow y = \frac{-1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}$

$\therefore m_2 = \frac{-1}{\sqrt{3}}$

Let θ be the angle between the lines. Then,

$$\tan\theta = \left|\frac{-\sqrt{3}+\frac{1}{\sqrt{3}}}{1+(-\sqrt{3})\left(\frac{-1}{\sqrt{3}}\right)}\right| = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{\sqrt{3}} \times \frac{1}{2}\right| = \left|\frac{-1}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$\tan\theta = \tan 30^\circ$ and $\tan(180^\circ - 30^\circ)$

$\Rightarrow \theta = 30^\circ$ and 150°

88. The line through the points (h, 3) and (4, 1) intersects the line 7x - 9y - 19 = 0 at right angle. Find the value of h.

Ans. : Slope of the line passing through the points (h, 3) and (4, 1) is

$= \frac{1-3}{4-h} = \frac{-2}{4-h}$

Also slope of the line 7x - 9y - 19 = 0 is $\frac{7}{9}$

Since two lines are perpendicular to each other

$\therefore \frac{-2}{4-h} \times \frac{7}{9} = -1 \Rightarrow \frac{-14}{36-9h} = -1 \Rightarrow$ -14 = -36 + 9h

$\Rightarrow$ 9h = 36 - 14 $\Rightarrow h = \frac{22}{9}$

89. Prove that the line through the point (x_1, y_1) and parallel to the line Ax + By + C = 0 is $A(x - x_1) + B(y - y_1) = 0$.

Ans. : Equation of the line parallel to line Ax + By + C = 0 is Ax + By + K = 0 . . . (i)

Since line (i) passes through (x_1, y_1)

$Ax_1 + By_1 + K = 0$...(ii)

Subtracting (ii) from (i), we have

$A(x - x_1) + B(y - y_1) = 0$

90. Find the values of θ and p, if the equation $x\cos\theta + y\sin\theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$

Ans. : Here $\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2 \Rightarrow -\sqrt{3}x - y = 2$

Dividing both sides by $\sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$, we have

$\frac{-\sqrt{3}}{2}x - \frac{1}{2}y = 1$

Put $\cos\alpha = \frac{-\sqrt{3}}{2}$ and $\sin\alpha = \frac{-1}{2}$

$\Rightarrow \alpha$ lies in 3rd quadrant

$\therefore \cos\alpha = \frac{-\sqrt{3}}{2} = -\cos30^\circ$ = cos (180° + 30°)

$\Rightarrow \alpha = 210^\circ$

$\therefore$ Equation of line in normal form is

$x\cos\frac{7\pi}{6} + y\sin\frac{7\pi}{6} = 1$

Comparing it with x cos $\alpha + y\ sin\alpha = p$, we have

$\alpha = \frac{7\pi}{6}$ and p = 1

91. What are the points on the y-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Ans. : Let point on y-axis be (0, y).

The given equation of line is $\frac{x}{3} + \frac{y}{4} = 1.$

$\Rightarrow 4x + 3y = 12 \Rightarrow 4x + 3y - 12 = 0$

Now perpendicular distance from point (0, y) to line 4x + 3y - 12 = 0 is

$$\left|\frac{4\times0+3y-12}{\sqrt{(4)^2+(3)^2}}\right| = \left|\frac{3y-12}{\sqrt{25}}\right| = \left|\frac{3y-12}{5}\right|$$

It is given that

$$\left|\frac{3y-12}{5}\right| = 4 \Rightarrow \left|\frac{3y-12}{5}\right| = \pm 4$$

When $\frac{3y-12}{5} = 4 \Rightarrow 3y - 12 = 20 \Rightarrow y = \frac{32}{3}$

When $\frac{3y-12}{5} = -4 \Rightarrow 3y - 12 = -20 \Rightarrow y = \frac{-8}{3}$

Thus required points are $\left(0, \frac{32}{3}\right)$ and $\left(0, \frac{-8}{3}\right)$.

92. Find the value of p so that three lines 3x + y - 2 = 0, px + 2y - 3 = 0 and 2x - y - 3 = 0 may intersect at one point.

Ans. : The equation of lines are
3x + y - 2 = 0, px + 2y - 3 = 0 and 2x - y - 3 = 0.
We know that three lines are concurrent if
$a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$
$\therefore 2[1 \times (-3) - 2 \times (-2)] + (-1)[-2$
$\times p - (-3) \times 3] + (-3)[3 \times 2 - p \times 1] = 0$
$\Rightarrow 2[-3 + 4] - 1[-2p + 9] - 3[6 - p] = 0$
$\Rightarrow 2 + 2p - 9 - 18 + 3p = 0$
$\Rightarrow 5p - 25 = 0 \Rightarrow p = 5.$

93. Find the equation of the line, which makes intercepts –3 and 2 on the x- and y-axes respectively.

Ans. : Here a = –3 and b = 2.

We know that equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-3} + \frac{y}{2} = 1$ or 2x - 3y + 6 = 0

94. Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with positive direction of x-axis is 15°.

Ans. : We are given that, p = 4 and ω = 15^0

Now, $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

and $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

The equation of the line is x cos ω + y sin ω = p
$x\cos 15^\circ + y\sin 15^\circ$
or $\frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = 4$
or $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$
This is the required equation.

95. The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K = 273 when F = 32 and that K = 373 when F = 212. Express K in terms of F and find the value of F, when K = 0

Ans. : Suppose that F along x-axis and K along y-axis, we have two points (32, 273) and (212, 373) in XY-plane. By two-point form, the point (F, K) satisfies the equation,then we get

$\mathrm{K} - 273 = \frac{373-273}{212-32}(\mathrm{F} - 32)$ or $\mathrm{K} - 273 = \frac{100}{180}(\mathrm{F} - 32)$

or $K = \frac{5}{9}(F - 32) + 273$ ....(1)

which is the required relation.

When K = 0, Equation (1) gives

$0 = \frac{5}{9}(\mathrm{F} - 32) + 273$ or $\mathrm{F} - 32 = -\frac{273 \times 9}{5} = -491.4$ or F= -459.4

96. Reduce the equation $\sqrt{3}x + y - 8 = 0$ into normal form. Find the values of p and ω

Ans. : We have,

$\sqrt{3}x + y - 8 = 0$ ... (1)

Dividing (1) by $\sqrt{(\sqrt{3})^2 + (1)^2} = 2$, we obtain

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$ or $\cos 30^\circ x + \sin 30^\circ y = 4$ ....(2)

Comparing (2) with x cos ω + y sin ω = p, then we get p = 4 and ω = 30°.

97. Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.

Ans. : Given lines are

$y - \sqrt{3}x - 5 = 0$ or $y = \sqrt{3}x + 5$ ....(1)

and $\sqrt{3}y - x + 6 = 0$ or $y = \frac{1}{\sqrt{3}}x - 2\sqrt{3}$......(2)

Slope of line (1) is $m_1 = \sqrt{3}$ and slope of line (2) is $m_2 = \frac{1}{\sqrt{3}}$

The acute angle (say) θ between two lines is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \text{......(3)}$$

Substituting,the values of m_1 and m_2 in (3), we obtain

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right| = \left|\frac{1-3}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

which gives θ = 30°.

Therefore,the angle between two lines is either 30°or 180° – 30° = 150°.

98. Find the equation of a line perpendicular to the line x - 2y + 3 = 0 and passing through the point (1, – 2).

Ans. : Given line x - 2y + 3 = 0 can be written as

$y = \frac{1}{2}x + \frac{3}{2}$ ....(1)

Slope of the line (1) is $m_1 = \frac{1}{2}$. Thus,slope of the line perpendicular to line (1) is

$m_2 = -\frac{1}{m_1} = -2$

Equation of the line with slope – 2 and passing through the point (1, – 2) is

$y - (-2) = -2(x - 1)$ or $y = -2x$

This is the required equition of line

99. If the lines 2x + y - 3 = 0, 5x + ky - 3 = 0, 3x - y - 2 = 0 are concurrent, find the value of k.

Ans. : Given that three lines are concurrent, if they pass through a common point, i.e., point of intersection of any two lines lies on the third line. Here given lines are

2x + y – 3 = 0 ....(1)

5x + ky – 3 = 0 ....(2)

3x – y – 2 = 0 ... (3)

Solving (1) and (3) by cross-multiplication method, we get $\frac{x}{-2-3} = \frac{y}{-9+4} = \frac{1}{-2-3}$ or x = 1, y = 1

Therefore, the point of intersection of two lines is (1, 1).
Since the above three lines are concurrent, the point (1, 1) will satisfy equation (2) so that

5.1 + k.1 – 3 = 0

$\Rightarrow$ k = – 2

100. By using the concept of slope, show that the points (-2, -1), (4, 0), (3, 3) and (-3, 2) are the vertices of a parallelogram.

Ans. : Let the vertices be A(-2, -1), B(4, 0), C(3, 3) and D(-3, 2).

Using slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, we get:

Slope of $AB(m_1) = \frac{0-(-1)}{4-(-2)} = \frac{1}{6}$

Slope of $CD(m_2) = \frac{2-3}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$

$\Rightarrow m_1 = m_2 \Rightarrow AB||CD$

Also

Slope of $AD\ (m_2) = \frac{2-(-1)}{-3-(-2)} = \frac{3}{-1} = -3$

Slope of $BC\ (m_4) = \frac{3-0}{3-4} = \frac{3}{-1} = -3$

$\Rightarrow m_3 = m_4 \Rightarrow AD||BC$

Hence, ABCD is a parallelogram.

101. Find the equation of the line on which the length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x-axis is 30°.

Ans. : Given inclination of perpendicular line (L) passing through origin is 30°

$\Rightarrow$ Slope $= \tan 30^\circ = \frac{1}{\sqrt{3}}$

Slope of perpendicular line (M) which is perpendicularn to line L is $-\sqrt{3}$

So equation of line M is $y = -\sqrt{3x} + c$

Given perpendicular distance from origin to line M is 4

$4 = \frac{c}{2} \Rightarrow c = 8$

So, equation of line M is $y = -\sqrt{3x} + 8$

102. Reduce the equation $\sqrt{3}x + y + 2 = 0$ to:
Intercept form and find intercept on the axes;

Ans. : Intercept from $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2$

$\Rightarrow \frac{\sqrt{3}x}{-2} + \frac{y}{-2} = 1$

$\Rightarrow \frac{x}{\frac{-2}{\sqrt{3}}} + \frac{y}{-2} = 1$

$\Rightarrow$ x intercept $= \frac{-2}{\sqrt{3}}$, y intercept = -2

103. State whether the two lines in the following are parallel, perpendicular or neither.
Through (5, 6) and (2, 3); through (9, -2) and (6, -5)

Ans. : Slope of line joining (5, 6) and (2, 3)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{2-5} = \frac{-3}{-3} = 1$$

Slope of line joining (9, -2) and (6, -5)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-2)}{6-9} = \frac{-5+2}{-3} = 1$$

Here $m_1 = m_2$

$\therefore$ The two lines are parallel.

104. Reduce the equation $\sqrt{3}x + y + 2 = 0$ to:
slope-intercept form and find slope and y-intercept;

Ans. : Slope intercept from (y = mx + c)

$\sqrt{3}x + y + 2 = 0$

$\Rightarrow y = -\sqrt{3}x - 2$

$\Rightarrow m = -\sqrt{3}, \ c = -\sqrt{2}$

y-intercept = -2, slope = $-\sqrt{3}$

105. Without using Pythagoras theorem, show that the points A(0, 4), B(1, 2) and C(3, 3) are the vertices of a right angled triangle.

Ans. : Slope of $AB = \frac{2-4}{1-0} = -2$

Slope of $BC = \frac{3-2}{3-1} = \frac{1}{2}$

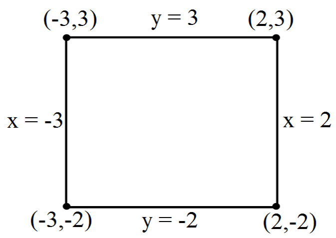
slope of AB × slope of BC $= -2 \times \frac{1}{2} = -1$

$\therefore$ Anlgle between AB and BC $= \frac{\pi}{2}$

$\therefore$ ABC are the vertices of a right angled triangle.

106. Draw the lines x = -3, x = 2, y = -2, y = 3 and write the coordinates of the vertices of the square so formed.

Ans. : The figure with the line $x = -3, x = 2, y = -2, y = 3$ is as follows:



From the figure, the coordinates of the vertices of the square are $(2, 3), (-3, 3), (-3, -2), (2, -2)$.

107. A straight line passes through the point $(\alpha, \ \beta)$ and this point bisects the portion of the line intercepted between the axes. Show that the equation of the straight line is $\frac{x}{2\alpha} + \frac{y}{2\beta} = 1$.

Ans. : The line intercepted by the axes are (a, 0) and (0, b), if this line segment is bisected at point $(\alpha, \ \beta)$ then $\frac{a+0}{2} = \alpha, \ \frac{0+y}{2} = \beta$ (using mid point formula)

$a = 2\alpha, b = 2\beta$

The equation of straight line in the intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2\alpha} + \frac{y}{2\beta} = 1$$

108. The equation of the line that passes through P (x_1, y_1) and makes an angle of θ with the x-axis is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$.

Ans. : The equation of line is

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$$

or

$x = x_1 \pm r\cos\theta$ and $y - y_1 = \pm r\sin\theta$

$Q(x_1 \pm r\cos\theta,\ y_1 \pm r\sin\theta)$ lie in $ax + by + c = 0$

$\Rightarrow\ a(x_1 + r\cos\theta) + b(y_1 \pm r\sin\theta) + c = 0$

$\Rightarrow\ \pm r(a\cos\theta + b\sin\theta) = -c - ax_1 - by_1$

$\Rightarrow\ -r = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$

109. If the straight line through the point P (3, 4) makes an angle $\frac{\pi}{6}$ with the x-axis and meets the line 12x + 5y + 10 = 0 at Q, find the length PQ.

Ans. : The equation of line is

$$\frac{x-3}{\cos\frac{\pi}{6}} = \frac{y-4}{\sin\frac{\pi}{6}} = \pm r$$

or $x = \pm\frac{\sqrt{3}}{2}r + 3$ and $y = \pm\frac{1}{2}r + 4$

$Q\left(\pm\frac{\sqrt{3}r}{2} + 3,\ \pm\frac{r}{2} + 4\right)$ lie in $12x + 5y + 10 = 0$

$\therefore 12\left(\pm\frac{\sqrt{3}r}{2} + 3\right) + 5\left(\pm\frac{r}{2} + 4\right) + 10 = 0$

$\pm\frac{12\sqrt{3}r}{2} + 36 \pm \frac{5r}{2} + 20 + 10 = 0$

$r = \frac{\pm 132}{5 + 12\sqrt{3}}$

Hence, lenght PQ is $\frac{132}{12\sqrt{3}+5}$

110. Prove that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices of a rectangle.

Ans. : Here A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3)

Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4+1}{-2+4}$

$M_{AB} = \frac{-3}{2}$

Slope of $BC = \frac{3+1}{2+4}$

$M_{BC} = \frac{2}{3}$

Slope of $AD = \frac{3+1}{2+4}$

$M_{AD} = \frac{2}{3}$

Slope of $CD = \frac{3-0}{2-4}$

$M_{CD} = \frac{-3}{2}$

$\Rightarrow M_{AB} = M_{CD}$ and $M_{BC} = M_{AD}$

$\Rightarrow AB || CD$ and $BC || AD$

$M_{AB} \times M_{BC} = \frac{-3}{2} \times \frac{2}{3}$

$M_{AB} \times M_{BC} = -1$

$AB \perp BC$

$M_{BC} \times M_{CD} = \frac{2}{3} \times \frac{-3}{2}$
$M_{BC} \times M_{CD} = -1$
$\Rightarrow BC \perp CD$
Thus,
$AB||CD$ and $BC||AD$
$AB \perp BC,\ BC \perp CD,\ CD \perp DA$
⇒ ABCD is a rectangle

111. Find the equation of a line for which:

$p = 8, \alpha = 300^\circ$

Ans. : $P = 8,\ \alpha = 300^\circ$
$x\cos\alpha + y\sin\alpha = P$
$\Rightarrow x\cos 300^\circ + y\sin 300^\circ = 8$
$\Rightarrow x \times \frac{1}{2} - y \times \frac{\sqrt{3}}{2} = 8$
$\Rightarrow x - \sqrt{3x} = 16$

*** Given section consists of questions of 3 marks each.** **[69]**

112. If p and q are the length of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\cos ec\theta = k$ respectively, prove that $p^2 + 4q^2 = k^2$.

Ans. : Length of perpendicular from origin to line $x\cos\theta - y\sin\theta - k\cos 2\theta = 0$ is

$$p = \left| \frac{0 \times \cos\theta - 0 \times \sin\theta - k\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} \right| = \left| \frac{-k\cos 2\theta}{1} \right| = k\cos 2\theta$$

Length of perpendicular from origin to line $x\sec\theta + y\cos ec\theta - k = 0$ is

$$q = \left| \frac{0 \times \sec\theta + 0 \times \cos ec\theta - k}{\sqrt{\sec^2\theta + \cos ec^2\theta}} \right| = \left| \frac{-k}{\sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}}} \right|$$

$= |-k\sin\theta\cos\theta| = \frac{k}{2}\sin 2\theta$

Now $p^2 + 4q^2 = (k\cos 2\theta)^2 + 4\left(\frac{k}{2}\sin 2\theta\right)^2$

$= k^2(\cos^2 2\theta + \sin^2 2\theta) = k^2.$

113. Find the perpendicular distance from the origin of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

Ans. : Equation of the line joining points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$
$\Rightarrow (\cos\phi - \cos\theta)y - \sin\theta\cos\phi + \sin\theta\cos\theta$
$= (\sin\phi - \sin\theta)x - \sin\phi\cos\theta + \sin\theta\cos\theta$
$\Rightarrow (\sin\phi - \sin\phi)x - (\cos\phi - \cos\theta)y - \sin\phi\cos\theta + \sin\theta\cos\phi = 0$

$\Rightarrow (\sin\phi - \sin\phi)x - (\cos\phi - \cos\theta)y + \sin(\theta - \phi) = 0$

Now perpendicular distance from (0, 0) to the given line is

$$= \left| \frac{(\sin\phi - \sin\theta)\times 0 - (\cos\phi - \cos\theta \times 0 + \sin(\theta-\phi)}{\sqrt{(\sin\phi - \sin\theta)^2 + (\cos\phi - \cos\theta)^2}} \right|$$

$$= \left| \frac{\sin(\theta-\phi)}{\sqrt{\sin^2\phi + \sin^2\theta - 2\sin\phi\sin\theta + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}} \right|$$

$$= \left| \frac{\sin(\theta-\phi)}{\sqrt{2 - 2(\cos\theta\cos\phi + \sin\theta\sin\theta)}} \right|$$

$$= \left| \frac{\sin(\theta-\phi)}{\sqrt{2[1-\cos(\theta-\phi)]}} \right| = \left| \frac{\sin(\theta-\phi)}{\sqrt{2\left[2\sin^2\left(\frac{\theta-\phi}{2}\right)\right]}} \right|$$

$$= \frac{|\sin(\theta-\phi)|}{\left|2\sin\left(\frac{\theta-\phi}{2}\right)\right|}.$$

114. Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point where it meets the Y-axis.

Ans. : Given equation of line is

$\frac{x}{4} + \frac{y}{6} = 1 \Rightarrow \frac{3x+2y}{12} = 1$

$\Rightarrow$ 3x + 2y = 12 ...(i)

If line (i) meet the Y-axis, then put x = 0 in Eq. (i), we get

0 + 2y = 12 $\Rightarrow$ y = 6

$\therefore$ Point is (0, 6).

Slope of line (i) is, $m_1 = \frac{-3}{2}$

$\therefore$ Slope of line perpendicular to line (i) is,

$m_2 = -\frac{1}{m_1} = \frac{-1}{(-3/2)} = \frac{2}{3}$

Now, equation of line having slope $\frac{2}{3}$ and passing through (0, 6) is given by

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 6 = \frac{2}{3}(x - 0)$

$\Rightarrow$ 3y - 18 = 2x

$\Rightarrow$ 2x - 3y + 18 = 0

which is required equation of line.

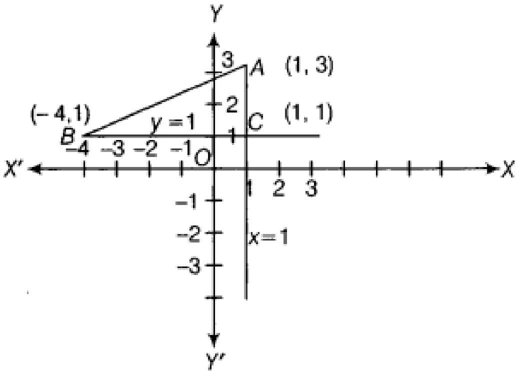
115. The hypotenuse of a right-angled triangle has its ends at the points (1, 3) and (-4, 1). Find the equation of the legs (perpendicular sides) of the triangle.

Ans. :

First we plot the points A(1, 3) and B (-4, 1) in the XY-plane. From the point A(1, 3), we draw a line parallel to Y-axis. And the point B(-4, 1), we draw a line parallel to X-axis. The point of intersection of two lines is on C, which is right angled at C.

$\therefore$ The coordinate of C will be (1, 1)

∴ Equation of line AC passing through A(1, 3) and C(1, 1) is



$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}\left(x - x_1\right)$

∴ $y - 3 = \frac{1-3}{1-1}(x - 1)$

⇒ $y - 3 = \frac{-2}{0}(x - 1) \Rightarrow x = 1$

Equation of line BC is

$y - 1 = \frac{1-1}{1+4}(x - 1)$

⇒ $y - 1 = \frac{0}{1+4}(x - 1)$

⇒ y - 1 = 0 ⇒ y = 1

Hence, the legs of a triangle are x = 1 and y = 1.

116. A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

Ans. : Let BA be the incident ray and AC be the reflected ray.

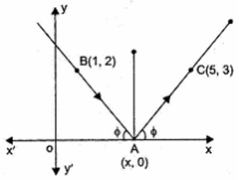
Now for line AC

$\tan\phi = \frac{3-0}{5-x}$

$\Rightarrow \tan\phi = \frac{3}{5-x}$....... (i)

Now for line BA

$\tan(180 - \phi) = \frac{2-0}{1-x}$



⇒ $-\tan\phi = \frac{2}{1-x}$. . . (ii)

From (i) and (ii), we have

$\frac{3}{5-x} = \frac{-2}{1-x}$ ⇒ 3 - 3x = - 10 + 2x ⇒ -5x = -13

$\Rightarrow x = \frac{13}{5}$

Thus coordinates of point A are $\left(\frac{13}{5}, 0\right)$

117. Prove that the product of the lengths of the perpendiculars drawn from the points $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

Ans. : Let P_1 and P_2 be the length of perpendiculars from $\left(\sqrt{a^2-b^2},0\right)$ and $\left(-\sqrt{a^2-b^2},0\right)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$.

$$\therefore P_1 = \left|\frac{\frac{\sqrt{a^2-b^2}\cos\theta}{a}+\frac{0\times\sin\theta}{b}-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

$$= \left|\frac{\frac{\sqrt{a^2-b^2}\cos\theta-1}{a}}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\right|$$

$$P_2 = \left|\frac{\frac{-\sqrt{a^2-b^2}\cos\theta}{a}+\frac{0\times\sin\theta}{b}-1}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\right|$$

$$= \left|\frac{\frac{-\sqrt{a^2-b^2}\cos\theta}{a}-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\right|$$

$$\text{Now } P_1P_2 = \left|\frac{\frac{\sqrt{a^2-b^2}\cos\theta}{a}-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\right|\left|\frac{\frac{-\sqrt{a^2-b^2}\cos\theta}{a}-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\right|$$

$$= \frac{\left|\left[\frac{\sqrt{a^2-b^2}\cos}{a}-1\right]\left[\frac{\sqrt{a^2-b^2}\cos\theta}{a}+1\right]\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|\left[\frac{(a^2-b^2)\cos^2\theta}{a^2}-1\right]\right|}{\frac{\cos^2\theta}{a^2}+\frac{1-\cos^2\theta}{b^2}} = \frac{\left|\left[\frac{(a^2-b^2)\cos^2\theta}{a^2}-1\right]\right|}{\frac{b^2\cos^2\theta+a^2-a^2\cos^2\theta}{a^2b^2}}$$

$$= \frac{\left|a^2-(a^2-b^2)\cos^2\theta\right|}{\frac{a^2-(a^2-b^2)\cos^2\theta}{b^2}}$$

$$= a^2-(a^2-b^2)\cos^2\theta\times\frac{b^2}{a^2-(a^2-b^2)\cos^2\theta}$$

$= b^2$.

118. Find the distance of the line 4x – y = 0 from the point P (4, 1) measured along the line making an angle of 135° with the positive x-axis.

Ans. :

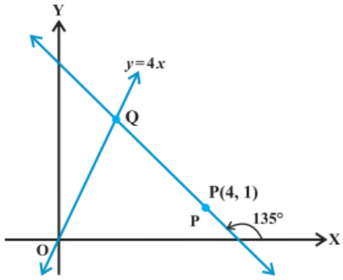
Given that the line is 4x – y = 0 ... (1)

In order to find the distance of the line (1) from the point P (4, 1) along another line, we have to find the point of intersection of both the lines.

For this purpose, we will first find the equation of the second line.

Slope of second line is tan 135° = –1.

Equation of the line with slope -1 through the point P (4, 1) is



y – 1 = – 1 (x – 4) or x + y – 5 = 0 ... (2)

Solving (1) and (2), we get x = 1 and y = 4 so that point of intersection of the two lines Q (1, 4).

Now, distance of line (1) from the point P (4, 1) along the line (2)

= the distance between the points P (4, 1) and Q (1, 4).

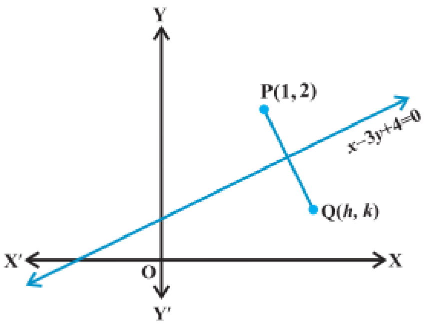
$= \sqrt{(1-4)^2 + (4-1)^2} = 3\sqrt{2}$ units.

Therefore, the required distance is $3\sqrt{2}$ units.

119. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1, 2) in the line x – 3y + 4 = 0

Ans. : Suppose Q (h, k) is the image of the point P (1, 2) in the line

x – 3y + 4 = 0 ... (1)



Thus,the line (1) is the perpendicular bisector of line segment PQ

Therefore,Slope of line PQ=$\frac{-1}{\text{Slope of line } x-3y+4=0}$

so that $\frac{k-2}{h-1} = \frac{-1}{\frac{1}{3}}$ or 3h+k=5

and the mid-point of PQ, i.e., point $\left(\frac{h+1}{2}, \frac{k+2}{2}\right)$ will satisfy the equation (1) so that

$\frac{h+1}{2} - 3\left(\frac{k+2}{2}\right) + 4 = 0$ or $h - 3k = -3$ ....(3)

Solving (2) and (3), we obtain $h=\frac{6}{5}$ and $k=\frac{7}{5}$

Therefore, the image of the point (1, 2) in the line (1) is $\left(\frac{6}{5}, \frac{7}{5}\right)$

120. Show that the area of the triangle formed by the lines $y = m_1 x + c_1$, $y = m_2x + c_2$ and $x = 0$ is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$

Ans. : Given lines are

$y = m_1x + c_1$ ... (1)

$y = m_2x + c_2$ ... (2)

$x = 0$ ... (3)

We know that line $y = mx + c$ meets the line $x = 0$ (y-axis) at the point (0, c). Thus, two vertices of the triangle formed by lines (1) to (3) are P $(0, c_1)$ and Q $(0, c_2)$. Third vertex can be obtained by solving equations (1) and (2). Solving (1) and (2), we obtain

$$x = \frac{(c_2 - c_1)}{(m_1 - m_2)} \text{ and } y = \frac{(m_1c_2 - m_2c_1)}{(m_1 - m_2)}$$

Thus, third vertex of the triangle is R $\left(\frac{(c_2 - c_1)}{(m_1 - m_2)}, \frac{(m_1c_2 - m_2c_1)}{(m_1 - m_2)}\right)$

Now, the area of the triangle is given

$$= \frac{1}{2}\left|0\left(\frac{m_1c_2 - m_2c_1}{m_1 - m_2} - c_2\right) + \frac{c_2 - c_1}{m_1 - m_2}(c_2 - c_1) + 0\left(c_1 - \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)\right| = \frac{(c_2 - c_1)^2}{2\left|m_1 - m_2\right|}$$

121. Find the length of the perpendicular from the origin to the straight line joining the two points whose coordinates are $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$.

Ans. : Line formed from joining $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$

$$\Rightarrow y - a\sin\beta = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha} \times x - \cos\beta$$

$$\Rightarrow y - a\sin\beta = \frac{2\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)}{-2\sin\left(\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)} \times (x - a\cos\beta)$$

$$\Rightarrow y - a\sin\beta = -\cot\left(\frac{\beta+\alpha}{2}\right)(x - a\cos\beta)$$

$$\Rightarrow y + \cot\left(\frac{\alpha+\beta}{2}\right)x - a - \cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta = 0$$

The, the length of perpendicular

$$\Rightarrow \left|\frac{0(y) + 0 - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta}{\sqrt{1 + \cot^2\left(\frac{\alpha+\beta}{2}\right)}}\right|$$

$$\Rightarrow \frac{a\cos\beta\cot\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta}{\operatorname{cosec}\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow a\cos\beta\cos\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\Rightarrow a\cos\left(\frac{\alpha-\beta}{2}\right) \ [\text{using } \cos A\cos B + \sin A\sin B = \cos(A\text{-}B)]$$

Hence, proved.

122. Find the equation of the straight line which passes through the point (-3, 8) and cuts off positive intercepts on the coordinate axes whose sum is 7.

Ans. : Let equation of line be

$\frac{x}{a} + \frac{y}{b} = 1$

then a + b = 7 and a ≥ 0 and b ≥ 0

$\therefore \frac{x}{a} = \frac{y}{7-a} = 1 \ldots (1)$

The line passes through (-3, 8)

$\Rightarrow \frac{-3}{a} + \frac{8}{7-a} = 1$

$\Rightarrow -21 + 3a + 8a = 7a - a^2$

$\Rightarrow -21 + 11a = 7a - a^2$

$\Rightarrow a^2 + 4A - 21 = 0$

⇒ A = 3 or -7

$a \neq -7 (\text{as } a \geq 0)$

∴ a = 3 and b = 4

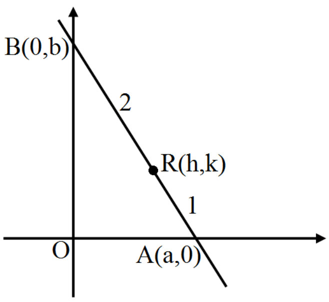
∴ Equation of line is

$\frac{x}{3} + \frac{y}{4} = 1$

or $4x + 3y = 12$

123. Point R (h, k) divides a line segment between the axes in the ratio 1 : 2. Find the equation of the line.

Ans. :



Point (h, k) divides the line segment in the ratio 1 : 2

Thus, using section point formula, we have

$h = \frac{2 \times a + 1 \times 0}{1 + 2}$

and

$k = \frac{2 \times 0 + 1 \times b}{1 + 2}$

Therefore, we have,

$h = \frac{2a}{3}$ and $k = \frac{b}{3}$

$\Rightarrow a = \frac{3h}{2}$ and $b = 3k$

Thus, the correspoinding point of A and B are $\left(\frac{3h}{2}, 0\right)$ and (0, 3k)

Thus, the equation of line joining the points A and B is

$\frac{y - 3k}{3k - 0} = \frac{x - 0}{0 - \frac{3h}{2}}$

$\Rightarrow -\frac{3h}{2}(y - 3k) = x \times 3k$

$\Rightarrow -3hy + 9hk = 6kx$

$\Rightarrow 2kx + hy = 3kh$

124. Find the equation of the straight line on which the length of the perpendicular from the origin makes an angle of 30° with x-axis and which forms a triangle of area $\frac{50}{\sqrt{3}}$ with the axes.

Ans. : $\alpha = 30^\circ$

Area of triangle = $\frac{50}{\sqrt{3}}$

Area of triangle = $\frac{1}{2}r^2\sin\theta = \frac{50}{\sqrt{3}}$

$\sin 30 = \frac{1}{2}$

$\frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = \frac{50}{\sqrt{3}}$

$p^2 = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 25$

$p \pm 5$

$x\cos\alpha + y\sin\alpha = \pm 5$

$x\cos 30^\circ + y\sin 30^\circ = \pm 5$

$x\frac{\sqrt{3}}{2} + \frac{y}{2} = \pm 5$

$\sqrt{3}x + y = \pm 10$

125. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle $\tan^{-1}(\frac{5}{12})$ with the positive direction of x-axis.

Ans. : Here $p = 3$

and $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\Rightarrow \cos\alpha = \frac{12}{13},\ \sin\alpha = \frac{5}{13}$

Equation of straight line is:

$x\cos\alpha + y\sin\alpha = p$

$x\left(\frac{12}{13}\right) + y\left(\frac{5}{13}\right) = 3$

$12x + 5y = 39$

126. Find the value of θ and p, if the equation $x\cos\theta + y\sin\theta = p$ is the normal form of the line $\sqrt{3}x + y + 2 = 0$.

Ans. : We have,

$\sqrt{3x} + y + 2 = 0$

$-\sqrt{3x} - y = 2$

$\left(-\frac{\sqrt{3}}{2}\right)x + \left(\frac{-1}{2}\right)y = 1$

This same as $x\cos\theta + y\sin\theta = p$

Therefore, $\cos\theta = \frac{-\sqrt{3}}{2},\ \sin\theta = -\frac{1}{2}$ and $p = 1$

$\theta = 210^\circ$ and $p = 1$

$\theta = \frac{7\pi}{6}$ and $p = 1$

127. Find the equation of the straight line which makes a triangle of area $96\sqrt{3}$ with the axes and perpendicular from the origin to it makes an angle of 30° with Y-axis.

Ans. : Perpendicular from origin makes an angle of 30° with y axis, thus making 60° worth x axis

Area of triangle is $= 96\sqrt{3}$

$\frac{1}{2} \times 2p \times \frac{2p}{\sqrt{3}} = 96\sqrt{3}$

$p^2 = \frac{96\sqrt{3} \times \sqrt{3}}{2} = 48 \times 3 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$p = 12$

$x\cos\alpha + y\sin\alpha = p$

$x\cos 60^\circ + y\sin 60^\circ = 12$

$x \times \frac{1}{2} + y\frac{\sqrt{3}}{2} = 12$

$x + \sqrt{3}y = 24$

128. Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.

Ans. : Given that: $\frac{x}{a} + \frac{y}{b} = 1 \ldots\ldots (i)$

and $\frac{x}{a} - \frac{y}{b} = 1 \ldots\ldots (ii)$

Slope of eq. (i) m_1 (say) = $-\frac{b}{a}$

And slope of eq. (ii) m_2 (say) = $\frac{b}{a}$

Let θ be the angle between the equation (i) and (ii)

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{-\frac{2b}{a}}{1 - \frac{b^2}{a^2}}\right| = \left|\frac{-2ab}{a^2 - b^2}\right|$$

$$\Rightarrow \tan\theta = \frac{2ab}{a^2 - b^2}.$$

Hence proved.

129. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$.

Ans. : Since p is the length of perpendicular fro m the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$,

We have

$$\Rightarrow p = \frac{0 + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{ab}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow p^2 = \frac{a^2 b^2}{a^2 + b^2}$$

Given that, a^2, p^2 and b^2 are in A.P.

$\therefore 2p^2 = a^2 + b^2$

$\Rightarrow \frac{2a^2b^2}{a^2+b^2} = a^2 + b^2$

$\Rightarrow 2a^2b^2 = (a^2 + b^2)^2$

$\Rightarrow 2a^2b^2 = a^2 + b^4 + 2a^2b^2$

$\Rightarrow a^4 + b^4 = 0$

130. For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x - 3y + 6 = 0 on the axes.

Ans. : The given equation are ax + by + 8 = 0 .....(i)

⇒ ax + by + 8 = 0

⇒ ax + by = -8

$\Rightarrow \frac{a}{-8}x + \frac{b}{-8}y = 1$

$\Rightarrow \frac{x}{\frac{-8}{a}} + \frac{y}{\frac{-8}{b}} = 1$

So, the intercepts ono the axes are $\frac{-8}{a}$ and $\frac{-8}{b}$

From eq. (ii), we get

⇒ 2x - 3y + 6 = 0

⇒ 2x - 3y = -6

$\Rightarrow \frac{2x}{6} - \frac{3y}{-6} = 1$

$\Rightarrow \frac{x}{-3} + \frac{y}{2} = 1$

So, the intercepts are -3 and 2.

According to the question

$\Rightarrow \frac{-8}{a} = +3$

$\Rightarrow a = -\frac{8}{3}$

and $\frac{-8}{b} = -2 \Rightarrow b = +4$

Hence, the required values of a and b are $\frac{-8}{3}$ and 4.

131. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinate axes such that their sum is 14.

Ans. : Equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$

Given that, a + b = 14 ⇒ b = 14 - a

So, equation of line is: $\frac{x}{a} + \frac{y}{14-a} = 1$

Since it passes through the point (3, 4), we have

$\Rightarrow \frac{3}{a} + \frac{4}{14-a} = 1$

⇒ a2 - 13a + 42 = 0

⇒ (a - 7)(a - 6) = 0

∴ a = 7, then b = 7

When a = 7, then b = 7

When a = 6, then b = 8

Thus, equation of line is: $\frac{x}{7} + \frac{y}{7} = 1,$

i.e., x + y = 7 or $\frac{x}{6} + \frac{y}{8} = 1$

132. Match the questions given under Column C_1 with their appropriate answers given under the Column C_2:

	Column C_1		Column C_2
(a)	The coordinates of the points P and Q on the line x + 5y = 13 which are at a distance of 2 units from the line 12x - 5y + 26 = 0 are	(i)	$(3, 1), (-7, 11)$
(b)	The coordinates of the point on the line x + y = 4, which are at a unit distance from the line 4x + 3y - 10 = 0 are	(ii)	$-\frac{1}{3}, \frac{11}{3}, \frac{4}{3}, \frac{7}{3}$
(c)	The coordinates of the point on the line joining A (-2, 5) and B (3, 1) such that AP = PQ = QB are	(iii)	$1, \frac{12}{5}, -3, \frac{16}{5}$

Ans. :

	Column C_1		Column C_2
(a)	The coordinates of the points P and Q on the line x + 5y = 13 which are at a distance of 2 units from the line 12x - 5y + 26 = 0 are	(iii)	$1, \frac{12}{5}, -3, \frac{16}{5}$
(b)	The coordinates of the point on the line x + y = 4, which are at a unit distance from the line 4x + 3y - 10 = 0 are	(i)	$(3, 1), (-7, 11)$
(c)	The coordinates of the point on the line joining A (-2, 5) and B (3, 1) such that AP = PQ = QB are	(ii)	$-\frac{1}{3}, \frac{11}{3}, \frac{4}{3}, \frac{7}{3}$

Solution:

1. Let $P(x_1, y_1)$ be any point of the given line

x + 5y = 13 $\therefore$ $x_1 + 5y_1 = 13$ Distance of line 12x - 5y + 26 = 0 from the point $P(x_1, y_1)$

$2 = \left|\frac{12x_1 - 5y_1 + 26}{\sqrt{(12)^2 + (-5)^2}}\right| \quad \Rightarrow 2 = \left|\frac{12x_1 - (13 - x_1) + 26}{13}\right|$

$\Rightarrow 2 = \left|\frac{12x - 1 - 13 + x_1 + 26}{13}\right| \quad \Rightarrow 2 = \left|\frac{13x_1 + 13}{13}\right| \quad \Rightarrow 2 = \pm(x_1 + 1)$

$\Rightarrow 2 = x_1 + 1 \Rightarrow x_1 = 1$ (Taking (+) sign) and $2 = -x_1 - 1 \Rightarrow x_1 = -3$ (Taking (-) sing)

Putting the values of x_1 in eq. $x_1 + 5y = 13$. We get $y_1 = \frac{12}{5}$ and $\frac{16}{5}$. So, the required points are $\left(1, \frac{12}{5}\right)$ and $\left(-3, \frac{16}{5}\right)$ Hence, (a) ⇔ (iii).

2. Let P (x_1, y_1) be any point on the given line

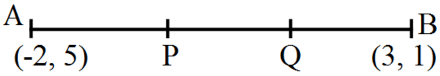
x + y = 4 $\therefore$ $x_1 + y_1 = 4$ .....(i) Distance of the line 4x + 3y - 10 = 0 from the point $P(x_1, y_1)$ $\quad \Rightarrow 1 = \left|\frac{4x_1 + 3y_1 - 10}{\sqrt{(4)^2 + (3)^2}}\right| \quad \Rightarrow 1 = \left|\frac{4x_1 + 3(4 - x_1) - 10}{5}\right|$

$\Rightarrow 1 = \left|\frac{4x_1 + 12 - 3x_1 - 10}{5}\right| \quad \Rightarrow 1 = \left|\frac{x_1 + 2}{5}\right| \quad \Rightarrow 1 = \pm\left(\frac{x_1 + 2}{5}\right) \quad \Rightarrow \frac{x_1 + 2}{5} = 1$

(Taking (+) sign) $\Rightarrow x_! + 2 = 5 \Rightarrow x_1 = 3$ and $\frac{x_1 + 2}{5} = -1$ (Taking (-) sign) $\Rightarrow x_1 + 2 = -5 \Rightarrow x_1 = -7$ Putting the values of x_1 in eq. (i) we get $x_1 + y_1 = 4$ at $x_1 = 3$, $y_1 = 1$ at $x_1 = -7$, $y_1 = 11$ So, the required are (3, 1) and B(-7, 11) Hence, (b) ⇔ (i).

3. Given that AP = PQ = QB

Equation of line joining A(-2, 5) and B(3, 1) is $y-5 = \frac{1-5}{3+2}(x+2)$



$\Rightarrow y-5 = \frac{-4}{5}(x+2) \Rightarrow 5y - 25 = -4x - 8 \Rightarrow 4x + 5y - 17 = 0$ Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points on the AB $P(x_1, y_1)$ divides the line the AB in the ratio 1 : 2 $\therefore x_1 = \frac{1.3+2(-2)}{1+2} = \frac{3-4}{3} = \frac{-1}{3}$ $y_1 = \frac{1.1+2.5}{1+2} = \frac{1+10}{3} = \frac{11}{3}$ So, the coordinates of $P(x_!, y_1) = \left(\frac{-1}{3}, \frac{11}{3}\right)$. Now point $Q(x_2, y_2)$ is the mid point of PB $\therefore x_2 = \frac{3-\frac{1}{3}}{2} = \frac{4}{3}$ $y_2 = \frac{1+\frac{11}{3}}{2} = \frac{7}{3}$ Hence, the coordinates of $Q(x_2, y_2) = \left(\frac{4}{3}, \frac{7}{3}\right)$

Hence, (c) ⇔ (ii).

133. Find the equation of lines passing through point $(0, a)$ on which the perpendicular drawn from the point $(2a, 2a)$ is of length a.

Ans. : Let the slope of line passing through point $(0, a)$ is m.

The equation of line $y-a = mx$

$\Rightarrow \quad mx - y + a = 0$.........(1)

According to question, the length of perpendicular drawn from point $(2a, 2a) = a$

$$\Rightarrow \quad \frac{|2am-2a+a|}{\sqrt{(m)^2+(-1)^2}} = a$$

$$\Rightarrow \quad \frac{|2am-a|}{\sqrt{m^2+1}} = a$$

$$\Rightarrow \quad \frac{|(2m-1)|}{\sqrt{m^2+1}} = 1$$

$$\Rightarrow \quad |(2m-1)| = \sqrt{m^2+1}$$

On squaring both sides,

$(2m-1)^2 = m^2+1$

$\Rightarrow \quad 4m^2 - 4m + 1 = m^2 + 1$

$\Rightarrow \quad 3m^2 - 4m = 0$

$\Rightarrow \quad m(3m-4) = 0$

$\Rightarrow \quad m = 0, m = \frac{4}{3}$

(i) When $m = 0$, then equation of line is required

$y - a = 0$[From equation (1)]

(ii) When $m = \frac{4}{3}$, then equation of the required line is

$\frac{4x}{3} - y + a = 0$

$\Rightarrow \quad 4x - 3y + 3a = 0$

134. Find the image of point (2, 3) w.r.t. line $x - 2y + 1 = 0$.

Ans. : Equation of given line AB (say)

$x - 2y + 1 = 0$........(1)

Image

If image of point (2, 3) w.r.t. to line $x - 2y + 1 = 0$ be $Q(h, k)$, then line $PQ \perp AB$ and mid-point of line PQ will be at line AB .

Now equation of line perpendicular to line AB is given by

$2x + y + k = 0$

$\because$ This line passes through point $P(2, 3)$. So this point will satisfy the equation of line. Hence,

$4 + 3 + k = 0 \Rightarrow k = \ -7$

So, equation of line PQ

$2x + y - 7 = 0$.......(2)

Coordinates of point R of intersection of lines (1) and

(2) = $\left(\frac{13}{5}, \frac{9}{5}\right)$

$\because$ Point R is the mid-point of line PQ .

So,

$\frac{h+2}{2} = \frac{13}{5}$ and $\frac{k+3}{2} = \frac{9}{5}$

$\Rightarrow \quad 5h + 10 = 26 \quad \Rightarrow \quad 5k + 15 = 18$

$\Rightarrow \quad 5h = 16 \quad \Rightarrow \quad 5k = 3$

$\Rightarrow \quad h = \frac{16}{5} \quad \Rightarrow \quad k = \frac{3}{5}$

Hence, required point of image $\left(\frac{16}{5}, \frac{3}{5}\right)$

*** Given section consists of questions of 5 marks each.** **[135]**

135. Find the equation of the line passing through the point of intersection of the lines 4x + 7y - 3 = 0 and 2x - 3y + 1 = 0 that has equal intercepts on the axis.

Ans. : The equation of given lines are 4x + 7y - 3 = 0 and 2x - 3y + 1 = 0.

Now the equation of any line through intersection of these lines is

4x + 7y - 3 + k(2x - 3y + 1) = 0. . . (i)

$\Rightarrow$ (1 + 2k)x + (7 - 3k)y = 3 - k

$\Rightarrow \frac{(4+2k)x}{3-k} + \frac{(7-3k)y}{3-k} = 1$

$\Rightarrow \frac{x}{\frac{3-k}{4-2k}} + \frac{y}{\frac{3-k}{7-3k}} = 1$

It is given that $\frac{3-k}{4+2k} = \frac{3-k}{7-3k}$

$\Rightarrow (3-k)\left[\frac{1}{4+2k} - \frac{1}{7-3k}\right] = 0$

$\Rightarrow$ 3 - k = 0 and $\frac{1}{4+2k} - \frac{1}{7-3k} = 0$

$\Rightarrow$ 3 = k or 7 - 3k - 4 - 2k = 0

$\Rightarrow$ k =3 or -5k = - 3

$\Rightarrow$ k = 3 or $k = \frac{3}{5}$

Putting k = 3 in (i) , we have

4x + 7y - 3 + 3(2x - 3y + 1) = 0

$\Rightarrow$ 4x + 7y - 3 + 6x - 9y + 3 = 0

$\Rightarrow$ 10x - 2y = 0 $\Rightarrow$ 5x - y = 0

Putting $k = \frac{3}{5}$ in (i), we have

$4x + 7y - 3 + \frac{3}{5}(2x - 3y + 1) = 0$

$\Rightarrow$ 20x + 35y - 15 + 6x - 9y + 3 = 0.

$\Rightarrow$ 26x + 26y - 12 = 0 $\Rightarrow$ 13x + 13y - 6 = 0.

136. Show that the equation of the line passing through the origin and making an angle θ with the line y = mx + c is $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

Ans. : Let m_1 be the slope of required line which passes through (0, 0).

Then equation of line is y - 0 = m_1 (x - 0) $\Rightarrow$ y = m_1x

Now θ is the angle between y = mx + c and y = m_1x

$$\therefore \tan\theta = \left|\frac{m_1 - m}{1 + m_1 m}\right| \Rightarrow \tan\theta = \pm\frac{m_1 - m}{1 + m_1 m}$$

$$\Rightarrow \tan\theta = \frac{m_1 - m}{1 + m_1 m} \text{or} \tan\theta = -\frac{m_1 - m}{1 + m_1 m}$$

$$\Rightarrow \tan\theta + m_1 m \tan\theta = m_1 - m \text{ or } \tan\theta + m_1 m\tan\theta = m - m_1$$

$$\Rightarrow m_1(1 - m\tan\theta) = m + \tan\theta \text{ or } m_1(1 + m\tan\theta) = m - \tan\theta$$

$$\Rightarrow m_1 = \frac{m + \tan\theta}{1 - m\tan\theta} \text{or } m_1 = \frac{m - \tan\theta}{1 + m\tan\theta}$$

$$\Rightarrow m_1 = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$$

Putting value of m_1 in (i), we have

$$y = \pm\frac{m + \tan\theta}{1 - m\tan\theta} \cdot x$$

$$\Rightarrow \frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}.$$

137. Find equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it an crossing the y-axis at a distance of 2 units below the origin.

Ans. : Here $m = \tan\frac{2\pi}{3}$ = tan 120° = tan (90 + 30) = -cot 30° = $-\sqrt{3}$

Equation of the line passing through point (0, 2) having slope $-\sqrt{3}$ is

y - 2 = $-\sqrt{3}$(x - 0) $\Rightarrow$ $\sqrt{3}$ + y - 2 = 0

Now the line parallel to this line has slope $-\sqrt{3}$

Here c= -2

Putting these values in y = mx + c, we have

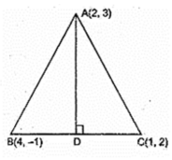
y = $-\sqrt{3}$ - 2 $\Rightarrow$ $-\sqrt{3}$ - y - 2 = 0

138. In the triangle ABC with vertices A(2, 3), B (4, -1) and C (1, 2) find the equation and length of altitude from the vertex A.

Ans. : Slope of $BC = \frac{2-(-1)}{1-4} = \frac{2+1}{-3} = \frac{3}{-3} = -1$

Since AD$\perp$BC, so slope of AD = 1.

$\therefore$ Equation of AD is



y - 3 = 1 (x - 2) $\Rightarrow$ x - y + 1 = 0

Equation of line BC is

y + 1 = -1(x - 4) $\Rightarrow$ x + y - 3 = 0

$$\therefore \text{ Length of } AD = \left|\frac{2+3-3}{\sqrt{(1)^2+(1)^2}}\right| = \left|\frac{2}{\sqrt{2}}\right| = \sqrt{2} \text{ units}$$

139. The equation of the base of an equilateral triangle is x + y = 2 and its vertex is (2, -1). Find the length and equations of its sides.

Ans. : The slope of AB = -1

Let slope of AC be m

Then,

$\tan 60^\circ = \frac{m+1}{1-m}$

$m = 2 - \sqrt{3}$

And similarly slope of $AB = 2 + \sqrt{3}$.

Equation of AC and AB are

$(y+1) = (2 - \sqrt{3})(x - 2)$

or, $(2 - \sqrt{3})\ x - y - 5 + 2\sqrt{3} = 0\ ...\ (i)$

and,

$(y - 1) = (2 + \sqrt{3})(x - 2)$

or, $(2 + \sqrt{3})\ x - y - 5 - 2\sqrt{3} = 0\ ...\ (ii)$

On solving (i) with x + y = 2, we get

$A\left(\frac{21 - 11\sqrt{3}}{6}, \frac{11\sqrt{3} - 9}{6}\right)$

$AB = AC = BC$

$= \sqrt{\left(\frac{21 - 11\sqrt{3} - 1}{6}\right)^2 + \left(\frac{11\sqrt{3} - 9 - 1}{6}\right)^2}$

$= \sqrt{\frac{225 + 363 - 330\sqrt{3} + 363 + 225 - 330\sqrt{3}}{36}}$

$= \sqrt{\frac{2}{3}}$

140. Find the equations to the sides of an isosceles right angled triangle the equation of whose hypotenues is 3x + 4y = 4 and the opposite vertex is the point (2, 2).

Ans. : Let the isosceles right triangle be.

AC = 3x + 4y = 4

C(2, 2)

Then, slope of $AC = \frac{-3}{4}$

AB = BC $\left[\therefore\ \text{It is an isoscales right triangle}\ \right]$

Then, angle between (AB and AC) and (BC and AC) is 45°.

$\tan\frac{\pi}{4} = \frac{m_1 - \left(\frac{-3}{4}\right)}{1 + \left(\frac{-3}{4}\right)m_1}$ [when m_1 = slope of BC]

$1 = \frac{m_1 + \frac{3}{4}}{1 - \frac{3}{4}m}$

$4 - 3m_1 = 4m_1 + 3$

$7m_1 = 1$

$m_1 = \frac{1}{7}$

and, $AB \perp BC$

$\therefore$ (Slope of AB) × (slope of BC) = -1

$m_2 \times \frac{1}{7} = -1$

$m_2 = -7$

The equation of BC is

$(y - 2) = \frac{1}{7}(x - 2)$

$7y - 14 = x - 2$
$x - 7y + 12 = 0$
and
The equation of AB is
$(y - 2) = -7(x - 2)$
$y - 2 = -x + 14$
$y + 7x - 16 = 0$

141. Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

Ans. : Let the image of p(3, 8) in x + 3y = 7 be Q(α, β)
Then.
PQ is perpendicular bisected at R.
Then,
$R = \left(\frac{\alpha+3}{2}, \frac{\beta+8}{2}\right)$
and lie on x + 3y = 7
$\frac{\alpha+3}{2} + \frac{3\beta+24}{2} = 7$
$\alpha + 3 + 3\beta + 24 = 14$
$\alpha + 3\beta = -13 \ldots (1)$
And since PQ is perpendicular to
x + 3y = 7
(Slope of line) × (Slope of PQ) = -1
$\frac{-1}{3} \times \frac{\beta-8}{\alpha-3} = -1$
$\beta - 8 = 3\alpha - 9$
$\beta - 3\alpha = -1 \ldots (2)$
Solving (1) and (2)
$\beta = -4, \alpha = -1$
$\therefore$ Q is (-1, -4)

142. Find the equations of two straight lines passing through (1, 2) and making an angle of 60° with the line x + y = 0. Find also the area of the triangle formed by the three lines.

Ans. : AC and BC are inclided to (AB) x + y = 0 at an angle of 60°
$\therefore \Delta ABC$ is equilateral triangle.
The slope of AB is -1 and let slope of AC be m_1
$\tan 60^\circ = \frac{m_1+1}{1-m_1}$ or $\sqrt{3}(1 - m_1) = m_1 + 1$
$\sqrt{3} - 1 = m_1 + \sqrt{3}m$
$\Rightarrow m_1 = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$
and, slope of BC is m_2
$\tan 60^\circ = \frac{m_2-1}{1+m_1} = \sqrt{3}$
$\therefore m_2 = \sqrt{3} + 2$
$\therefore$ Equation of AC and BC are
$y - 2 = (2 - \sqrt{3})(x - 1) \ldots (i)$
$y - 2 = (2 + \sqrt{3})(x - 1)$
using (i) and x + y = 0

A is $\left(\frac{-1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$A

AC is $\sqrt{\left(\frac{2+1+\sqrt{3}}{2}\right)^2 + \left(\frac{3-\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9+3+6\sqrt{3}+9+3-6\sqrt{3}}{4}}$

$\mathrm{AC} = \sqrt{\frac{24}{6}}$

$= \sqrt{6}$

The area of $\Delta\mathrm{ABC}$

$= \frac{\sqrt{3}}{4}(\mathrm{AC})^2$

$= \frac{\sqrt{3}}{4} \times (\sqrt{6})^2$

$= \frac{3}{2}\sqrt{3}$ sq untis.

143. Prove that the following sets of three lines are concurrent:

$\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1$ and $y = x$.

Ans. : $\frac{x}{a} + \frac{y}{b} = 1, \frac{x}{b} + \frac{y}{a} = 1$ and $y = x$

bx + ax = ab, ax + by = ab

put y = x

bx + ax = ab, ax + bx = ab

Hence the lines are concurrent

144. If the image of the point (2, 1) with respect to the line mirror be (5, 2), find the equation of the mirror.

Ans. : Let Q(5, 2) be the mirror image of P(2, -1) with respect to the line mirror AB × (ax + by + c = 0)

Then,

(Slope of AB) × (Slope of PQ) = -1

$\frac{-a}{b} \times \left(\frac{2-1}{5-2}\right) = -1$

$\frac{-a}{b} \times \frac{1}{3} = -1$

$-a = -3b$

$a = 3b \ldots (1)$

and

(R) mid point of PQ should line in AB, as PQ perpendicularlly bisects AB.

$\therefore$ Coordinates of R are $\left(\frac{5+2}{2}, \frac{2+1}{2}\right) = \left(\frac{7}{2}, \frac{3}{2}\right)$

$\therefore \frac{7}{2}a + \frac{3}{2}b + c = 0$

$7a + 3\left(\frac{a}{3}\right) + 2c = 0 \left[\because b = \frac{a}{3} \text{ from}(1)\right]$

$8a + 2c = 0$

or, $-4a = 6 \ldots (2)$

$\therefore$ equation of line is ax + by + c = 0

or, $ax + \frac{a}{3}y - 4a = 0$

or, $3x + y - 12 = 0$

145. Find the projection of the point (1, 0) on the line joining the points (-1, 2) and (5, 4).

Ans. : Let AB be the line, A = (-1, 2), B = (5, -4)

Then, equation of line AB is

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 2 = \frac{4 - 2}{5 + 1}(x + 1)$

$y - 2 = \frac{2}{6}(x + 1)$

$3y - x = 7 \ldots (1)$

Slope $= \frac{1}{3}$.

Let P point (1, 0) be the given point

Let Q(x_1, y_1) be the projection of P

Slope of PQ = -3 $[PQ \perp AB, m_1 m_2 = -1]$

Eq of PQ,

$y - 0 = -3(x - 1)$

$y = -3x + 3 \ldots (2)$

Solving (1) and (2)

$3y - \left(\frac{y - 3}{-3}\right) = 7$

$-9y - y + 3 = -21$

$-10y = -24$

$\Rightarrow \frac{12}{5} = -3x + 3$

$-3x = +\frac{12}{5} - 3 = \frac{+12 - 15}{5} = \frac{-3}{5}$

$x = \frac{1}{5}$

$\therefore N\left(\frac{1}{5}, \frac{'12}{5}\right)$

146. Show that the area of the triangle formed by the lines $y = m_1x$, $y = m_2x$ and $y = c$ is equal to $\frac{c^2}{4}(\sqrt{33} + \sqrt{11})$, , where m_1, m_2 are the roots of the equation $x^2 + (\sqrt{3} + 2)x + \sqrt{3} - 1 = 0$.

Ans. : $y = m_1x$, $y = m_2x$ and $y = c$

Vertices of triangle formed by above lines are

$A(0, 0);\ B\left(\frac{c}{m_1}, c\right);\ C\left(\frac{c}{m_2}, c\right)$

So Area of triangle when three vertices are given is

$\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

$= \frac{1}{2}\left[\left|\frac{c^2}{m_1} - \frac{c^2}{m_2}\right|\right] = \frac{c^2}{2}\left[\left|\frac{m_2 - m_1}{m_1 m_2}\right|\right]$

Given m_1 and m_2 are roots of $x^2 + (\sqrt{3} + 2)x + \sqrt{3} - 1 = 0$

Product of roots $= m_1 m_2 = \sqrt{3} - 1$

$|m_2 - m_1| = \sqrt{(m_2 + m_1)^2 - 4m_1m_2} = \sqrt{(\sqrt{3} + 2)^2 - 4\sqrt{3} + 4}$

$|m_2 - m_1| = \sqrt{3 + 4 + 4\sqrt{3} - 4\sqrt{3} + 4} = \sqrt{11}$

Area $= \frac{c^2}{2}\left[\frac{\sqrt{11}}{\sqrt{3} - 1}\right]$

Rationalising denominator gives $\frac{c^2}{4}[\sqrt{3}3 + \sqrt{1}1]$

Hence proved

147. Prove that the family of lines represented by $x(1 + \lambda) = y(2 - \lambda) + 5 = 0$, λ being arbitrary, pass through a fixed point. Also, find that point.

Ans. : $x(1 + \lambda) + y(2 - \lambda) + 5 = 0$

$\Rightarrow x + x\lambda + 2y - \lambda y + 5 = 0$

$\Rightarrow \lambda(x - y) + (x + 2y + 5) = 0$

$\Rightarrow (x + 2y + 5) + \lambda(x - y) = 0$

This is of the form $L_1 + \lambda L_2 = 0$

So it represents a line passing through the intersection of x - y = 0 and x + 2y = -5.

Solving the two equations, we get $\left(\frac{-5}{3}, \frac{-5}{3}\right)$ which is the fixed point through which the given family of lines passes for any value of λ.

148. If a, b, c are in A.P., prove that the straight lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent.

Ans. : If a, b, c are in A.P.

b - a = c - b

2b = a + c [Common difference]

To prove that the straight lines are concurent then they ihave the common point of intersection.

ax + 2y + 1 = 0 ...(1)

bx + 3y + 1 = 0 ...(2)

cx + 4y + 1 = 0 ...(3)

Solving (1) and (2)

$x = \frac{-1-2y}{a}$

Put in (2)

$b = \left(\frac{-1-2y}{a}\right) + 3\ y + 1 = 0$

$y = \frac{b-a}{3a-2b} \Rightarrow x = \frac{-1-\frac{2(b-a)}{3a-2b}}{a} = \frac{-3a+2b-2b+2a}{a(3a-2b)}$

$x = \frac{-1}{3a-2b}$

Putting x, y in (3)

$c\left(\frac{-1}{3a-2b}\right) + 4\left(\frac{b-a}{3a-2b}\right) + 1 = 0$

$-c + 4b - 4a + 3a - 2b = 0$

$-a + 2b - c = 0$

$-a + a + c - c = 0$

$0 = 0$

149. The line through (h, 3) and (4, 1) intersects the line 7x - 9y - 19 = 0 at right angle. Find the value of h.

Ans. : If two lines intersect at right angles, then product of their slope is -1.

Slope of 7x - 9y -19 = 0 is $m_1 = \frac{7}{9} \ldots (1)$

Slope of line joining (h, 3) and $(4, 1) = \frac{1-3}{4-h}$

or, $m_2 = \frac{2}{h-4} \ldots (2)$

$m_1 \times m_2 = -1$

$\frac{7}{9} \times \frac{2}{h-4} = -1$

$14 = -9h + 36$

$9h = 36 - 14$

$h = \frac{22}{9}$

150. Find the equation of the bisector of angle A of the triangle whose vertices are A (4, 3), B(0, 0) and C (2, 3).

Ans. : Let AD be the bisector of $\angle A$

Then, BD : DC=AB : AC

Now,

$|AB| = \sqrt{(4-0)^2 + (3-0)^2} = 5$

$|AC| = \sqrt{(4-2)^2 + (3-3)^2} = 2$

$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{2}$

⇒ D divides BC in the ratio 5 : 2

So the coordinates of D are $\left(\frac{5\times2+0}{5+2}, \frac{5\times3+0}{5+2}\right) = \left(\frac{10}{7}, \frac{15}{7}\right)$

∴ The equation of AD is

$y-3 = \left(\frac{\frac{15}{7}-3}{\frac{10}{7}-4}\right)(x-4)$

$y-3 = \left(\frac{15-21}{10-28}\right)(x-4)$

$\Rightarrow y - 3 = \frac{1}{3}(x-4)$

$\Rightarrow 3(y-3) = x - 4$

$\Rightarrow x - 3y + 9 - 4 = 0$

$\Rightarrow x - 3y + 5 = 0$

151. Find the angles between the following pairs of straight lines:

$(m^2 - mn)y = (mn + n^2)x + n^3$ and $(mn + m^2)y = (mn - n^2)x + m^3$.

Ans. : Converting the equation in the form

$y = mx + c$

$y = \frac{(mn+n^2)}{m^2-mn}x + \frac{n^3}{(m^2-mn)}$

$\Rightarrow m_1 = \frac{mn+n^2}{m^2-mn}$

Also, $y = \frac{(mn-n^2)}{mn+m^2}x + \frac{m^3}{mn+m^2}$

$\Rightarrow m_2 = \frac{mn-n^2}{mn+m^2}$

Thus, angle between 2 lines is $\tan\theta$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{\left(\frac{mn+n^2}{m^2-mn}\right)-\left(\frac{mn-n^2}{mn+m^2}\right)}{1+\left(\frac{mn+n^2}{m^2-mn}\right)\left(\frac{mn-n^2}{mn+m^2}\right)}\right|$

$= \left|\frac{m^2n^2+m^3n+n^3m+n^2m^2-m^3n+m^2n^2+n^2m^2-mn^3}{m^3n+m^4-m^2n^2-m^3n+m^2n^2-mn^3+mn^3-n^4}\right|$

$= \left| \frac{4m^2n^2}{m^4 - n^4} \right|$

$\Rightarrow \theta = \tan^{-1} \left| \frac{4m^2n^2}{m^4 - n^4} \right|$

152. Find the equation of the straight lines passing through the following pair of points:

$(at_1, \frac{a}{t_1})$ and $(at_2, \frac{a}{t_2})$

Ans. : Let $A(x_1y_1)$ be $\left(at_1, \frac{a}{t_1}\right)$

$B(x_2y_2)$ be $\left(at_2, \frac{a}{t_2}\right)$

Then equation of line AB is

$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - \frac{a}{t_1} = \frac{\frac{a}{t_2} - \frac{a}{t_1}}{at_2 - at_1}(x - at_1)$

$\Rightarrow y - \frac{a}{t_1} = \frac{a(t_1 - t_2)}{at_1t_2(t_2 - t_1)}(x - at_1)$

$\Rightarrow y - \frac{a}{t_1} = \frac{-1}{t_1t_2}(x - at_1)$

$\Rightarrow t_1t_2y + x = a(t_1 + t_2)$

$\therefore$ The equation of the line joining the points $\left(at_1, \frac{a}{t_1}\right)$ and $\left(at_2, \frac{a}{t_2}\right)$ is $t_1t_2y + x = a(t_1 + t_2)$

153. Prove that the perpendicular drawn from the point (4, 1) on the join of (2, -1) and (6, 5) divides it in the ratio 5:8.

Ans. : Let the perpendicular drawn from P(4, 1) on the line joining A(2, -1) and B(6, 5) divides in the ratio k : 1 at the point R.

Using section formula, coordinates of R are:

$x = \frac{6k+2}{k+1}$ and $y = \frac{5k-1}{k+1}$ . . . (i)

PR prependicular to AB

$\therefore$ (slope of PR) × (slope of AB)= -1

$\Rightarrow \left(\frac{y-1}{x-4}\right) \times \left(\frac{5-(-1)}{6-2}\right) = -1$

$\Rightarrow \frac{\frac{5k-1}{k+1} - 1}{\frac{6k+2}{k+1} - 4} \times \frac{6}{4} = -1$

$\Rightarrow \frac{5k-1-k-1}{6k+2-4k-4} = \frac{-4}{6}$

$\frac{4k-2}{2k-2} = \frac{-2}{3}$

$3(2x - 1) = -2(k - 1)$

$\Rightarrow 6k - 3 = -2k + 2$

$8k = 5$

$\Rightarrow k = \frac{5}{8}$

ratio is 5 : 8

$\therefore$ R divides ABin the ratio 5 : 8

154. If the length of the perpendicular from the point (1, 1) to the line ax - by + c = 0 be unity, show that $\frac{1}{c} + \frac{1}{a} - \frac{1}{b} = \frac{c}{2ab}$.

Ans. : Length of perpendicular from (1, 1) to ax - by + c = 0

$$\Rightarrow \left|\frac{a(1)-b(1)+c}{\sqrt{a^2+b^2}}\right| = 1$$

$a-b+c = \sqrt{a^2+b^2}$

$(a-b+c)^2 = a^2+b^2$

$a^2+b^2+c^2+2ac-2bc-2ab = a^2+b^2$

$c^2+2ac-2bc = 2ab$

$c+2a-2b = \frac{2ab}{c}$

$\frac{c}{2ab}+\frac{2a}{2ab}-\frac{2b}{2ab} = \frac{1}{c}$

$\frac{c}{2ab} = \frac{1}{c}+\frac{1}{a}-\frac{1}{b}$

Hence, prove

155. The perpendicular distance of a line from the origin is 5 units and its slope is -1. Find the equation of the line.

Ans. : The perpendicular distance from the origin to the line is 5, so

$x\cos\alpha + y\sin\alpha = 5$

$y\sin\alpha = -x\cos\alpha + 5$

$y = -\frac{\cos\alpha}{\sin\alpha}x + 5$

$y = -\cot\alpha x + 5$

Comparing with y = mx + c

$m = -\cot\alpha$

$-1 = -\cot\alpha$

$\cot\alpha = 1$

$\alpha = \frac{\pi}{4}$

So, the equation of line is

$x\cos\frac{\pi}{4} + y\sin\frac{\pi}{4} = 5$

$\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}} = 5$

$x+y+5\sqrt{2}$

156. Find the equation of a line which is perpendicular to the line $\sqrt{3}x - y + 5 = 0$ and which cuts off an intercept of 4 units with the negative direction of y-axis.

Ans. : Required equation of line is

$y - y_1 = m'(x - x_1)$ ...(1)

point is $(x_1y_1) = (0, -4)$

It is perpendicular to line $\sqrt{3}x - y + 5 = 0$

⇒ Slope is y = mx + c

$y = \sqrt{3}x + 5$

$m = \sqrt{3}$

$m' = \frac{-1}{m} = \frac{-1}{\sqrt{3}}$

Putting m' and (x_1y_1) in (1)

$y-(-4) = \frac{-1}{\sqrt{3}}(x-0)$

$y+4 = \frac{-x}{\sqrt{3}}$

$x+\sqrt{3}y + 4\sqrt{3} = 0$

157. Find the equations of the two straight lines through (1, 2) forming two sides of a square of which 4x + 7y = 12 is one diagonal.

Ans. : Let ABCD be a square whose diagnal BD is 4x + 7y = 12

Then, slope of $\mathrm{BD} = \frac{-4}{7}$

Let slope of AB = m

Then, $\tan 45^\circ = \frac{m+\frac{4}{7}}{1-\frac{4}{7}m}$

$7 - 4m = 7m + 4$

$11m = 3$

$\therefore m = \frac{3}{11}$

$\therefore$ slope of $\mathrm{BC} = \frac{-1}{\text{slope of AB}}$

$= \frac{-11}{3}$

$\therefore$ Equation of AB is

$(y-2) = \frac{3}{11}(x-1)$

$11y - 22 = 3x - 3$

$3x - 11y + 19 = 0$

and

Equation of BC is

$(y-2) = \frac{-11}{3}(x-1)$

$11x + 3y - 17 = 0$

158. Find the values of α so that the point $p(\alpha^2, \alpha)$ lies inside or on the triangle formed by the lines x - 5y + 6 = 0, x - 3y + 2 = 0 and x - 2y - 3 = 0.

Ans. : Let ABC be the triangle of the equations whose sides AB, BC and CA are respectively x - 5y + 6 = 0, x - 3y + 2 = 0 and x - 2y - 3 = 0

The coordinates of the vertices are A(9, 3), B(4, 2) and C(13, 5).

If the point $p(\alpha^2, \alpha)$ lies n side the $\triangle ABC$, THEN

i. A and P must be on the same side of BC.
ii. B and P must be on the same side of AC.
iii. C and P must be on the same side of AB.

Now,

A and P must be on the same side of BC if,

$\left(9(1) + 3(-3) + 2\right)\left(\alpha^2 - 3\alpha + 2\right) > 0$

$(9 - 9 + 2)\left(\alpha^2 - 3\alpha + 2\right) > 0$

$\alpha^2 - 3\alpha + 2 > 0$

$(\alpha - 1)(\alpha - 2) > 0$

$\alpha \in (-\infty, 1) \cup (2, \infty) \ldots (i)$

B and P must be on the same side of AC if,

$\left(13(1) + 5(-5) + 6\right)\left(\alpha^2 - 5\alpha + 6\right) > 0$

$\Rightarrow (-6)\left(\alpha^2 - 5\alpha + 6\right) > 0$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0$

$\Rightarrow (\alpha - 2)(\alpha - 3) < 0$

$\Rightarrow \alpha \in (2, 3) \ldots (ii)$

C and P must be on the same side of AB if,

$\left(4(1) + 2(-2) - 3\right)\left(\alpha^2 - 2\alpha - 3\right) > 0$

$(-3)\left(\alpha^2 - 2\alpha - 3\right) > 0$

$\alpha^2 - 2\alpha - 3 < 0$

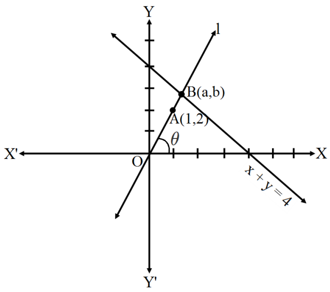
$(\alpha - 3)(\alpha + 1) < 0$

$\Rightarrow \alpha \in (-1, 3) \ldots (iii)$

From i, ii, and iii

$\alpha \in [2, 3]$

159. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line x + y = 4 is at a distance $\frac{\sqrt{6}}{3}$ from the given point.



Ans. :

Let the given line x + y = 4 and required line 'l' intersect at B(a, b).

Slope of line 'l' is given by $m = \frac{b-2}{a-1} = \tan\theta \ldots\ldots (i)$

Givne that $AB = \frac{\sqrt{6}}{3}$

So, by distance formula for point A(1, 2) and B(a, b), we get

$\sqrt{(a-1)^2 + (b-2)^2} = \frac{\sqrt{6}}{3}$

On squaring both the side

$a^2 + -2a + b^2 + 4 - 4b = \frac{6}{9}$

$a^2 b^2 - 2a - 4b + 5 = \frac{2}{3} \ldots\ldots (ii)$

Point B(a, b) also satisfies the eqn. x + y = 4

$\therefore a + b \ldots\ldots (iii)$

On solving (ii) and (iii), we get $a = \frac{3\sqrt{3}+1}{2\sqrt{3}}, b = \frac{5\sqrt{3}-1}{2\sqrt{3}}$

Putting values of a and b in eqn. (i), we have

$$\tan\theta = \frac{\frac{5\sqrt{3}-1}{2\sqrt{3}}}{\frac{3\sqrt{3}+1}{2\sqrt{3}}} = \frac{5\sqrt{3}-1-4\sqrt{3}}{3\sqrt{3}+1-2\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$\therefore \tan\theta = \tan 15^\circ$

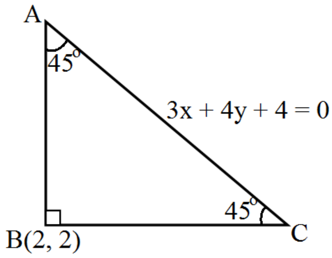
$\Rightarrow \theta = 15^\circ$

160. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by 3x + 4y = 4 and the opposite vertex of the hypotenuse is (2, 2).

Ans. : As shown in the figure, hypotenuse is along the line 3x + 4y + 4 = 0.

$\therefore$ Slope of AC $= \frac{-3}{4}$.

Since ABC is isosceles righta angled triangle,



Now, let the slope of the line making an angle 45° with AC be m.

$$\therefore \tan 45^\circ = \left| \frac{m - \left(-\frac{3}{4}\right)}{1 + m\left(-\frac{3}{4}\right)} \right|$$

$\Rightarrow \frac{4m+3}{4-3m} = \pm 1$

$\Rightarrow 4m + 3 = 4 - 3m$ or $4m + 3 = 3m - 4$

$\Rightarrow m = \frac{1}{7}$ or m = -7

So, if the slope of line BC is $\frac{1}{7}$ then the slope of line AB is -7.

So, equation of BC is: $y - 2 = \left(\frac{1}{7}\right)(x - 2)$

⇒ x - 7y + 12 = 0.

Equation of AB is: y - 2 = -7(x - 2)

⇒ 7x + y - 16 = 0.

161. P_1, P_2 are points on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P1 , P2 on the bisector of the angle between the given lines.

[**Hint:** Lines are $y = \sqrt{3}x + 2$ and $y = -\sqrt{3}x + 2$ according as $x \geq 0$ or $x < 0$. y-axis is the bisector of the angles between the lines. P_1, P_2 are the points on these lines at a distance of 5 units from the point of intersection of these lines which have a point on y-axis as common foot of perpendiculars from these points. The y-coordinate of the foot of the perpendicular is given by $2 + 5\cos 30^\circ$.]

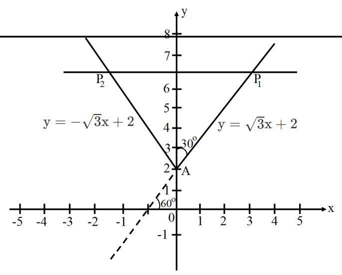
Ans. : Given lines are: $y - \sqrt{3}x = 2,\ \text{for } x \geq 0 \ldots\ldots \text{(i)}$

and $y + \sqrt{3}x = 2,\ \text{for } x \leq 0 \ldots\ldots \text{(ii)}$

Clearly, lines intersect ar A (, 2).

Line (i) is inclined at an angle of 60° with +ve direction of x-axis.

Line (ii) is inclined at an angle of 120° with +ve direction of x-axis.



P_1 and P_2 are points at distance 5 units from point A on the lines.
Clearly, angle bisector of lines is y-axis.
Foot of perpendicular from P_1 and P_2 on y-axis is B.
Now, $AP_1 = 5$

$\therefore$ In $\triangle ABP_1, \frac{AB}{AP_1} = \cos 30^\circ$

$\therefore AB = \frac{5\sqrt{3}}{2}$

$\therefore OB = 2 + \frac{5\sqrt{3}}{2}$

So, the coordinates of the perpendicular are $\left(0, 2 + \frac{5\sqrt{3}}{2}\right)$.

----- the journey of thousands miles begins with a single step -----